TN Department of Education

Pre-K–K Numeracy

Participant Manual

Tennessee Department of Education | 2016 Regional Educator Summit

Welcome, Participants!

We're excited to welcome you to this summer's **Regional Educator Summit**. We're impressed by your desire for professional learning and growth, and we hope you find this course productive and inspiring. As you engage in this training content over the next two days, we hope you make many connections to your own classroom practice. We look forward to hearing about the ways you implement this course content in the upcoming school year!

We are also proud to share that the content of this training was developed **by Tennessee educators, for Tennessee educators**. We believe it's important for professional development to be informed by current educators, who work in schools with students daily.

In particular, we'd like to thank the following educators who contributed to the creation and review of this content:

Content Developers

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- Kimberly Osborne - Murfreesboro City
- Heather Drinnon - Hamblen County

Module 4: Teaching Numbers in Base 10

Appendix

Introduction

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Supporting PreK-2 Educators

Beginning in 2012, the Tennessee Department of Education offered summer trainings with a focus on both the early foundational skills in math and on teaching mathematics content standards and practice standards through high-level instructional tasks.

The materials for the previous trainings can be accessed and downloaded at http://edutoolbox.org/tntools/menu/grade/803/955

Our Focus in 2016

The focus of the 2016 Regional Educator Summits for numeracy is to continue standards-driven learning that will support all students in achieving mathematical proficiency.

With rigor, this training will explore and analyze effective strategies to foster each child's growth of number sense and development of mathematical understanding within a numeracy-rich classroom. It will explicitly connect how to integrate literacy skills into mathematics instruction and learning.

Overall Goals

- Teachers will strengthen their knowledge of the Standards for Mathematical Practice and characteristics of mathematically proficient students.
- Teachers will learn to deepen students' understanding of number sense by implementing effective math instructional strategies within a numeracy-rich environment.
- Teachers will strengthen their own knowledge of the progression of numeracy skills within the CC, NBT, and OA domains of the Tennessee Academic Standards.
- Teachers will make important connections to literacy with a particular emphasis on incorporating speaking and listening in mathematics instruction.

Math in the Early Grades

"Early math abilities are a surprisingly important predictor of children's long-term success. A child's math ability when he or she enters school has proved a better predictor of academic achievement, high school graduation, and college attendance than any other early childhood skill. Mathematics learning is closely tied to students' executive function skills—a set of cognitive processes, including problem solving, reasoning, working memory, and task flexibility—that supports student achievement across academic subjects."

A. Szekely. *Unlocking Young Children's Potential: Governors' Role in Strengthening Early Mathematics Learning*. (Washington, D.C.: National Governors Association Center for Best Practices, October 28, 2014.)

Is it developmentally appropriate?

"Many people believe there is a developmental stage students must go through before they are ready for certain mathematical topics. But these ideas are outdated, as students are as ready as the experiences they have had, and if students are not ready, they can easily become so with the right experiences, high expectations from others, and a growth mindset."

"Students may be unready for some mathematics because they still need to learn some foundational, prerequisite mathematics they have not yet learned, but not because their brain cannot develop the connections because of their age or maturity."

Boaler, Jo. *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages, and Innovative Teaching*. (2016). San Francisco, CA: Jossey-Bass

Brain Research

"No one is born lacking the ability to learn math."

"Scientists now know that any brain differences present at birth are eclipsed by the learning experiences we have from birth onward."

"The new evidence from brain research tells us that everyone, with the right teaching and messages can be successful in math, and everyone can achieve at the highest levels in school."

"By including conceptual math teaching and encouraging a growth mindset in our classrooms, students will learn to shed the harmful ideas that they either "get it" or they don't."

"Students with a growth mindset take on hard work, and they view mistakes as a challenge and motivation to do more."

"The very best opportunities to learn come about when students believe in themselves."

(Boaler, 2016)

Video: Brain Plasticity—Jo Boaler

https://www.youtube.com/watch?v=pxru8H6XbR4&list=PLjzmW2yXs7pAPh2ek9CZwP-8w8VaGkC3Y&index=19

Stand Up, Hands Up, Pair Up

Stand up, raise your hand, find someone to pair up with, and answer the following questions:

- Has any information in the opening slides or videos been surprising to you?
- What is the early mathematics culture in your school or district?

Video on Math learning—Jo Boaler

https://www.youtube.com/watch?v=ZZrlk4NqaJ4

Talk with the people at your table and answer these questions:

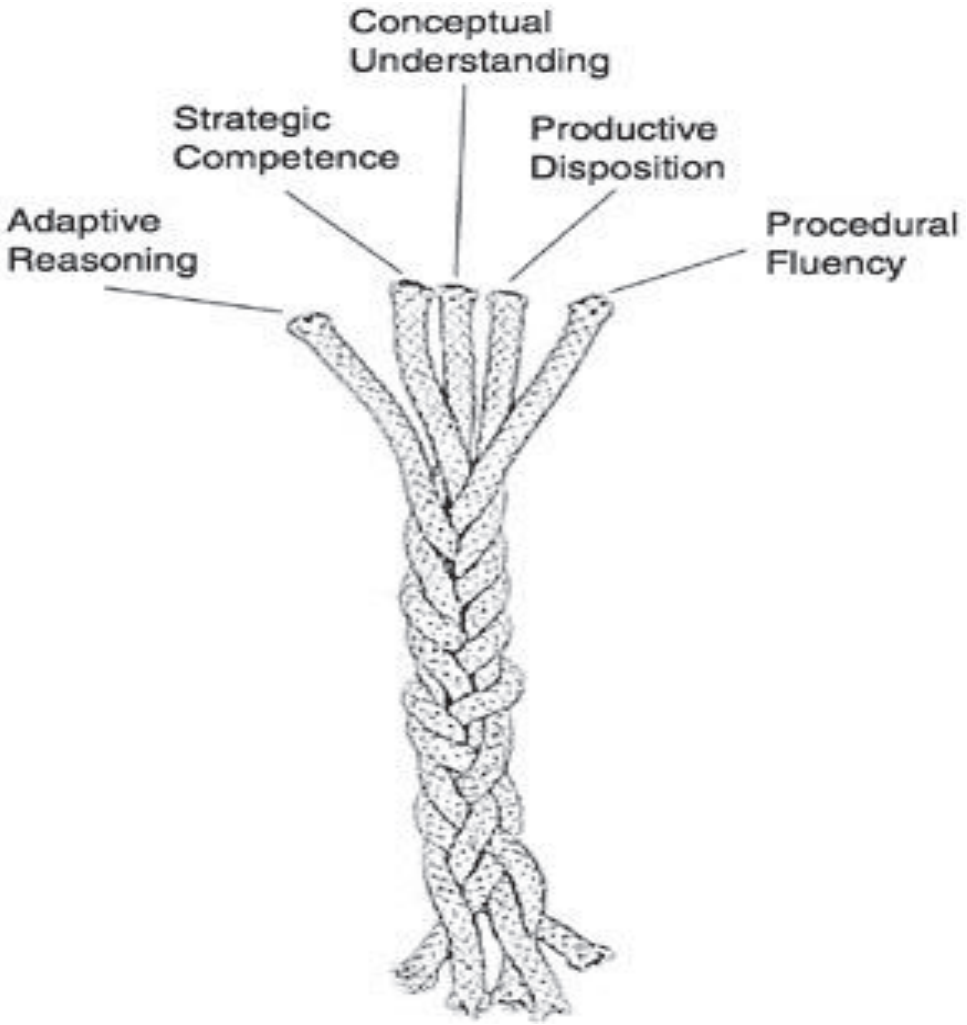
- ✓ Do you agree or disagree with her?

- ✓ How were you taught math?

- ✓ In what ways do you connect the math thinking of your students?

What is Mathematical Proficiency?

This “rope model” represents the five interwoven strands of mathematical proficiency.



National Research Council. (2001).
Adding it up: Helping children learn mathematics.

- **Conceptual understanding** is understanding mathematical concepts, operations, and relations.
- **Procedural fluency** is the ability to apply procedures accurately, efficiently, flexibly and appropriately.
- **Strategic competence** is the ability to formulate, represent, and solve mathematical problems.

- **Adaptive reasoning** is the capacity for logical thought, reflection, explanation and justification.
- **Productive disposition** is the habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

National Research Council. (2001).
Adding it up: Helping children learn mathematics.

In alignment with the five strands of mathematical proficiency, the Tennessee State Math Standards focus on a balanced development of conceptual understanding, procedural fluency, and application.

In addition, the Standards for Mathematical Practice are interwoven into the language of the content standards to reflect the proficiency strands of adaptive reasoning, strategic competence, and productive dispositions.

Retrieved from: https://www.tn.gov/assets/entities/sbe/attachments/1-29-16_II_A_Math_Standards_Attachment.pdf

Literacy Skills for Mathematical Proficiency

It is crucial that we recognize and develop the literacy skills that are necessary for achieving mathematical proficiency (i.e., reading, vocabulary, speaking, listening, writing).

Literacy skills for mathematical proficiency include:

- using multiple reading strategies
- understanding and using correct mathematical vocabulary
- discussing and articulating mathematical ideas
- communicating mathematical arguments

Mathematically proficient students

Mathematically proficient students do—

- **explain** to themselves the meaning of a problem and look for entry points to its solution.
- **make sense** of quantities and their relationships in problem situations.
- **use** assumptions, definitions, and previously established results in constructing arguments.
- **apply** the mathematics they know to solve problems arising in everyday life, society, and the workplace.
- **consider** all available tools when solving a mathematical problem.
- **communicate** precisely to others.
- **look closely** to discern a pattern or structure.
- **notice** if calculations are repeated and look for general methods and shortcuts.

Mathematically proficient students do not strictly—

- **copy** notes word for word from the board.
- **memorize** procedures for solving problems and duplicate exact replicas for homework and assessments.
- **sit** quietly throughout the entire lesson with minimal peer interaction.

Retrieved from http://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/What-Makes-a-Mathematically-Proficient-Student_/

Characteristics of Mathematically Proficient Students

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. (Common Core State Standards for Mathematics, p. 6)

Standards for Mathematical Practice	Characteristics of Mathematically Proficient Students
1. Make sense of problems and persevere in solving them.	**Mathematically proficient students can** • Explain the meaning of a problem and restate it in their words. • Analyze given information to develop possible strategies for solving the problem. • Identify and execute appropriate strategies to solve the problem. • Evaluate progress toward the solution and make revisions if necessary. • Check for accuracy and reasonableness of work, strategy and solution. • Understand and connect strategies used by others to solve problems.
2. Reason abstractly and quantitatively.	**Mathematically proficient students can** • Translate given information to create a mathematical representation for a concept. • Manipulate the mathematical representation by showing the process considering the meaning of the quantities involved. • Recognize the relationships between numbers/quantities within the process to evaluate a problem. • Review the process for reasonableness within the original context. **Mathematically proficient students can** • Use observations and prior knowledge (stated assumptions, definitions, and previous established results) to make conjectures and construct arguments.

3. Construct viable arguments and critique the reasoning of others.	• Compare and contrast logical arguments and identify which one makes the most sense. • Justify (orally and in written form) the approach used, including how it fits in the context from which the data arose. • Listen, understand, analyze, and respond to the arguments of others. • Identify and explain both correct and flawed logic. • Recognize and use counterexamples to refine assumptions or definitions and dispute or disprove an argument.
4. Model with mathematics.	**Mathematically proficient students can** • Use a variety of methods to model, represent, and solve real-world problems. • Simplify a complicated problem by making assumptions and approximations. • Interpret results in the context of the problem and revise the model if necessary. • Choose a model that is both appropriate and efficient to arrive at one or more desired solutions.
5. Use appropriate tools strategically.	**Mathematically proficient students can** • Identify mathematical tools and recognize their strengths and weaknesses. • Select and use appropriate tools to best model/solve problems. • Use estimation to predict reasonable solutions and/or detect errors. • Identify and successfully use external mathematical resources to pose or solve problems. • Use a variety of technologies, including digital content, to explore, confirm, and deepen conceptual understanding.
6. Attend to precision.	**Mathematically proficient students can** • Understand symbols and use them consistently within the context of a problem. • Calculate answers efficiently and accurately and label them appropriately.

	• Formulate precise explanations (orally and in written form) using both mathematical representations and words. • Communicate using clear mathematical definitions, vocabulary, and symbols.
7. Look for and make use of structure.	**Mathematically proficient students can** • Look for, identify, and accept patterns or structure within relationships. • Use patterns or structure to make sense of mathematics and connect prior knowledge to similar situations and extend to novel situations. • Analyze a complex problem by breaking it down into smaller parts. • Reflect on the problem as a whole and shift perspective as needed.
8. Look for and express regularity in repeated reasoning.	**Mathematically proficient students can** • Recognize similarities and patterns in repeated trials with a process. • Generalize the process to create a shortcut which may lead to developing rules or creating a formula. • Evaluate the reasonableness of results throughout the mathematical process while attending to the details.

(*) Collaborative Project with Cedarburg, Franklin, Fox Point-Bayside, Grafton, Greendale, Kettle Moraine, Menomonee Falls, Oconomowoc, Pewaukee, Waukesha, & Whitefish Bay School Districts and CESA 1.

Retrieved from http://www.cesa2.org/STEM/Characteristics%20Of%20Mathematically%20Proficient%20Students.pdf

Group Activity

On chart paper, make a list of activities and strategies you use to encourage your students to use the Standards for Mathematical Practices. What do these look like in the early childhood classroom? Be prepared to share your charts with the group.

Module 1:

Making Sense of Number Sense

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Module 1: Making Sense of Number Sense

Teachers will learn to deepen students' understanding of number sense by implementing effective math instructional strategies within a numeracy-rich environment.

Objectives

- Gain an understanding of number sense in the early childhood classroom
- Learn how to create a numeracy rich environment
- Learn strategies and activities to teach subitizing and number representations

Standards

Know number names and the count sequence.

Pre-K Standards	Kindergarten Standards
PK.CC.3. Understand the relationships between numerals, names of numbers, and quantities up to 10 (includes subitizing—the ability to look at a quantity and say the quantity [one through four] quickly, just by looking).	**K.CC.2.** Count forward beginning from a given number within the known sequence (instead of having to begin at one).

TEAM Alignment

- Teacher Content Knowledge
- Activities and Materials
- Problem Solving

Discussion

- What do you think is important in an early childhood program? List your ideas below.

NCTM Position

Young learners' future understanding of mathematics requires an early foundation based on a high-quality, challenging, and accessible mathematics education. Young children in every setting should experience mathematics through effective, research-based curricula and teaching practices. Such practices in turn require that teachers have the support of policies, organizational structures, and resources that enable them to succeed in this challenging and important work.

Retrieved from http://www.nctm.org/Standards-and-Positions/Position-Statements/Mathematics-in-Early-Childhood-Learning/

- **How does the NCTM position match or differ from your thinking?**

Creating a Numeracy-Rich Environment

Read the following quote and think about your classroom. In what ways do you promote mathematical learning for your students?

"A numeracy rich environment promotes mathematical learning by students. Borrowing from the research regarding literacy education, where immersion in a literacy-rich environment is considered essential to promote learning, it is important that students are immersed in a world of mathematics. As students see numbers and math-related materials throughout the classroom and participate in real-world, meaningful problem-solving opportunities, they begin to see the connection mathematics has to their own lives. Mathematics is no longer solely problems in a textbook, but it becomes something to ponder."

Sammons, L. & Janis K. Drab Fackler, *Guided Math, A Framework for Mathematics Instruction*.

List your ideas here:

Vocabulary Activity

In order to create a numeracy-rich environment, we must first understand the key vocabulary in early childhood numeracy. These terms will be discussed over the course of this training:

- subitizing
- number representations
- five/10 frame
- decomposing numbers
- number talk
- think aloud
- part-part whole
- number sense

Teaching Strategies to Build Number Sense

Read the article below, *Understanding Number Sense*. After reading, divide into six groups. Each group will make a list of activities and strategies they use to teach one of the components.

Understanding Number Sense—Its Importance and Research-Based Teaching That Improve It

Retrieved from http://mathsolutions.com/making-sense-of-math/number-sense/understanding-number-sense/.

What Is Number Sense?

Number sense essentially refers to a student's "fluidity and flexibility with numbers," (Gersten & Chard, 2001). He/She has sense of what numbers mean, understands their relationship to one another, is able to perform mental math, understands symbolic representations, and can use those numbers in real world situations. In her book, "About Teaching Mathematics", Marilyn Burns describes students with a strong number sense in the following way: "[They] can think and reason flexibly with numbers, use numbers to solve problems, spot unreasonable answers, understand how numbers can be taken apart and put together in different ways, see connections among operations, figure mentally, and make reasonable estimates."

The National Council of Teachers in 1989 identified the following five components that characterize number sense:

- Number meaning
- Number relationships
- Number magnitude
- Operations involving numbers and referents for number
- Referents for numbers and quantities

Why Is Number Sense Important?

Number sense is important because it encourages students to think flexibly and promotes confidence with numbers—they "make friends with numbers" as Carlyle and Mercado charmingly refer to it in their book "Teaching Preschool and Kindergarten Math". Students come to understand that numbers are meaningful and outcomes are sensible and expected (Burns, 2007). Conversely, students who lack a strong number sense have trouble developing the foundation needed for even simple arithmetic much less more complex math.

In a recent study of 180 seventh graders conducted by the University of Missouri, researchers found that, "those who lagged behind their peers in a test of core math skills needed to function as adults were the same kids who had the least number sense or fluency way back when they started first grade." (Neergaard, 2013) This is particularly sobering when one considers that one in five U.S. adults lacks the math competency of a middle school student—leaving them unqualified for most jobs.

Teaching Strategies to Build Students' Number Sense

We know from a wide body of research that number sense develops gradually and over time resulting from an exploration of numbers, visualizing numbers in a variety of contexts, and relating to numbers in different ways. About Teaching Mathematics. A K-8 Resource, 3rd Edition, Marilyn Burns (2007) highlights the following key, research-based teaching strategies to build numbers sense:

Model different methods for computing.

When a teacher publicly records a number of different approaches to solving a problem–solicited from the class or by introducing her own—it exposes students to strategies that they may not have considered. As Marilyn Burns explains, "When children think that there is one right way to compute, they focus on learning and applying it, rather than thinking about what makes sense for the numbers at hand."

Ask students regularly to calculate mentally.

Mental math encourages students to build on their knowledge about numbers and numerical relationships. When they cannot rely on memorized procedures or hold large quantities in their heads, students are forced to think more flexibly and efficiently, and to consider alternate problem solving strategies. (Parrish, 2010)

Have class discussions about strategies for computing.

Classroom discussions about strategies help students to crystalize their own thinking while providing them the opportunity to critically evaluate their classmates' approaches. In guiding the discussion, be sure to track ideas on the board to help students make connections between mathematical thinking and symbolic representation (Conklin &

Sheffield, 2012). As noted in Classroom Discussions: Using Math Talk to Help Students Learn, the goal is "not to increase the amount of talk but the amount of high quality talk."

Make estimation an integral part of computing.
Most of the math that we do every day—deciding when to leave for school, how much paint to buy, what type of tip to leave in a restaurant, which line to get in at the grocery store relies not only on mental math but estimations. However traditional textbook rounding exercises don't provide the necessary context for students to understand estimating or build number sense. To do that, estimation must be embedded in problem situations.

Question students about how they reason numerically.
Asking students about their reasoning—both when they make mistakes AND when they arrive at the correct answer—communicates to them that you value their ideas, that math is about reasoning, and, most importantly, that math should make sense to them. Exploring reasoning is also extremely important for the teacher as a formative assessment tool. It helps her understand each student's strengths and weaknesses, content knowledge, reasoning strategies and misconceptions.

Pose numerical problems that have more than one possible answer.
Problems with multiple answers provide plenty of opportunities for students to reason numerically. It's a chance to explore numbers and reasoning perhaps more creatively than if there was "one right answer."

"Just as our understanding of phonemic awareness has revolutionized the teaching of beginning reading, the influence of number sense on early math development and more complex mathematical thinking carries implications for instruction" (Gersten & Chard, 2001).

Sources

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Parrish, Sherry. Number Talks: Helping Children Build Mental Math and Computation Strategies, Grades K-5. Sausalito, CA: Math Solutions, 2010. Print.

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A Month-to-month Resource. Sausalito, CA: Math Solutions, 2010. Print.

Way, Jenni. "Number Sense Series: Developing Early Number Sense."
University of Cambridge, NRICH (2013): n. pag. Web. 26 Aug. 2013.

Discussion

As a group, discuss and list different ways or activities you use to teach this strategy in your classroom.

Group 1: Model different methods for computing
Computing in pre-K and Kindergarten can encompass counting, comparing numbers, addition or subtraction, and problem solving. What are some ways that you model these skills for your students?

Group 2: Ask students regularly to calculate mentally
Asking students to use mental math helps them to formulate strategies. How do you ask your students to use mental math?

Group 3: Have class discussions about strategies for computing
Number Talks and Think Alouds are two strategies that can be used to help students discuss their mathematical thinking. What are some other ways you help students vocalize their thinking?

Group 4: Make estimation an integral part of computing.
Using an estimation jar and varying the objects can help students learn estimation skills. What are some other ways you use estimation in your classroom?

Group 5: Question students about how they reason numerically.
During calendar time, ask students to figure out what the date is for that day. Ask them how they came up with that number. One might say they counted to that number, another might say they knew what number came next on the calendar. Both students are showing their reasoning. How do you find out the reasoning of your students?

Group 6: Pose numerical problems that have more than one possible answer
During Morning Meeting, write the number 10 on the board. Ask students how they can make 10. Some might draw 10 pictures, some might write 5+5. They can show many possible answers. What do you do in your classroom to foster this type of thinking?

Independent Reflection

Think about the strategies you just read. Answer the following question:

- How do you foster the growth of number sense in your classroom on a daily basis?

- Based on what you've learned so far, what ideas do you plan to incorporate into your classroom?

How can we use Subitizing and Number Representations to foster Number Sense?

What is Subitizing?

Subitizing is the ability to understand "how many" without counting.

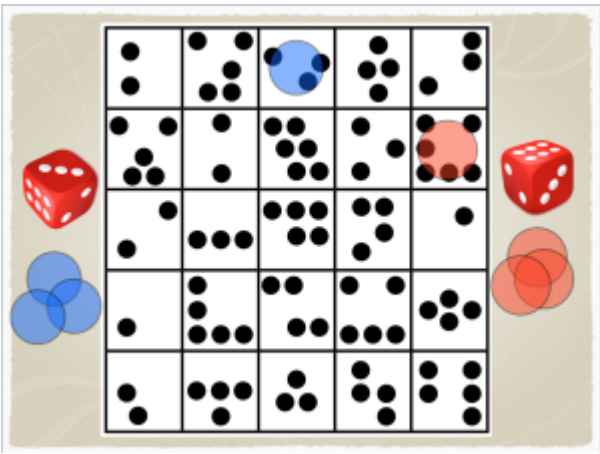
The ability to subitize is an important part of developing a strong mathematical foundation. There are two components of subitizing; conceptual and perceptual. Perceptual subitizing is the instant visual recognition of a pattern such as the dots on a die. Conceptual subitizing is recognizing smaller groups and adding them together, such as two dots plus two dots equals four dots. Subitizing can help children learn to:

- Develop estimation skills
- Count on from a known patterned set
- Combine numbers from sets
- Develop mathematical fluency

Retrieved from http://www.pre-kpages.com/subitizing/

Classroom Activities that Help to Develop Subitizing

- **Find-It Game:** You can download the game board from this site: https://mindfull.wordpress.com/2015/01/24/find-it-a-subitizing-bingo-game/.



- **On/Off Game:** Use a seasonal picture and matching manipulatives to play this game. In the picture you see an igloo and fish crackers. Decide on the number you want to work with and put that many manipulatives in a small cup. Students pour out the manipulatives on the work mat. They tell how many are "on" the mat, and how many are "off" the mat. **This is a fun partner game.**

- **Five frames/10 frames:** Use five frames first with dot stickers for subitizing practice. After students master the five frames, you can move into using 10 frames, and then double 10 frames for the teen numbers.

- **Finger games:** Say a number and have students hold up that many fingers. It is amazing how many students cannot do this. You hold up fingers for a few seconds and let them tell you how many you are holding up. You can also hold up some fingers for a few seconds then have them match you. This is a great activity to do while waiting in line for something.

- **Dominoes:** Students can sort dominoes by number.

- **Subitizing concentration game:** Print the game cards from the link below. Students match the cards.
  http://teachmath.openschoolnetwork.ca/wpcontent/uploads//2015/12/concentration1_k.pdf

- **Dot cards/plates**: Use plastic plates and colored dot stickers to make different groups and combinations. Hold up a plate for a few seconds for students to "see" how many.

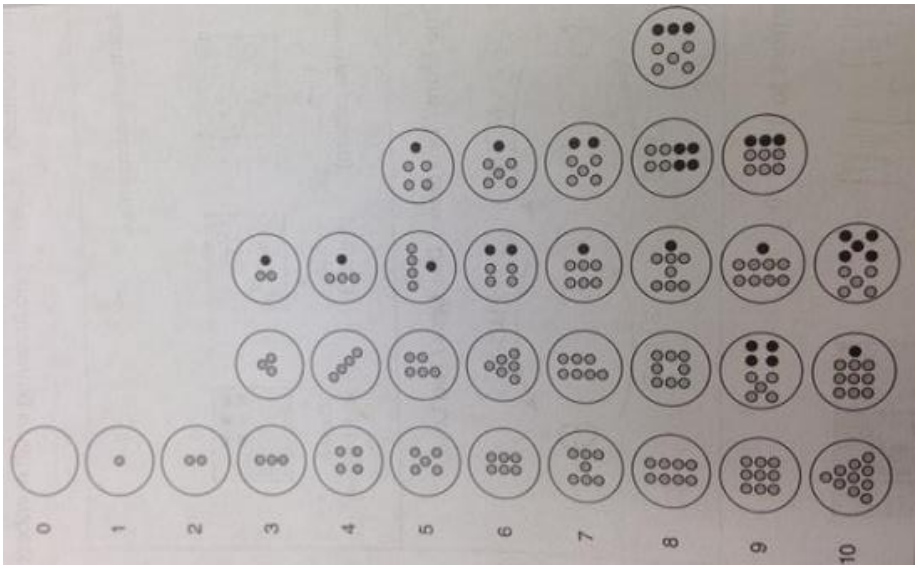
- **Doubles**: Teacher flashes a dot plate. Students respond with double the number of dots. The number range should reflect the grade level.

Think about the above activities. Talk with the people at your table and decide which of these activities show conceptual subitizing and which ones show perceptual subitizing. Do any of the activities show both types of subitizing? How do these activities foster Number Sense in the classroom?

Make and Take

Using the plates and dot stickers, make a set of dot plates you can use in your classroom. Think about your students' math knowledge at the beginning of the year, and make the plates so you can help build on that knowledge. We will be using these plates later in the training.

Dot plate examples



- Retrieved from
http://teachmath.openschoolnetwork.ca/wpcontent/uploads/documents/dotplatepatternsVDW.pdf

Video

https://www.teachingchannel.org/videos/visualizing-number-combinations

- Why is it helpful for students to discuss the mental images they formed?

- Ms. Latimer takes multiple student answers to the same question. What effect does this have?

- How could this lesson be connected with other skills?

Literature Connection:

Ten Black Dots by Donald Crews

This book uses colorful illustrations and rhymes to show common objects whose parts can be seen as various numbers of black dots. You can show the pages for three to four seconds, and then ask the children how many black dots they see. After learning the pattern of the book, have students make a prediction of how many dots will be on the next page, and what they are being used for.

The Very Hungry Caterpillar by Eric Carle

This book is great for students to see the "one more" pattern of numbers. On each day of the week, the caterpillar eats one more fruit than the day before. You can ask students to make predictions, or extend the learning by asking how much the caterpillar ate on Monday and Wednesday together.

Talk with the people at your table. List ideas you would like to use to support subitizing in your classroom below. Be ready to share.

What are Number Representations?

Number representations are ways to show numbers and their quantities. To help students develop an understanding of mathematical concepts, they need to understand that numbers can be represented in many different ways.

Classroom Activities for Number Representations:

- **Match-me games**: students can match different representations of numbers—quantities to numbers, tally marks to ten frames, 10 frame to words, etc.
- **Number of the day**: choose a number of the day and make multiple representations of it.
- **Numbers about Me:** Use a student's birthdate, age, etc. to make multiple representations.
- **Number posters**: Make different representations of numbers and let the students sort them onto the correct number poster (tally marks, 10 frame, numeral, number word, pictures).

Talk with the people at your table List ideas you would like to use to support number representations in your classroom below. Be ready to share.

Make and Take: Make a Number Poster

Using the template at the end of your manual, choose a number and make a number poster.

Show different ways that the number can be represented.

Number Sense Resources

- Number and Number Sense Module: http://www.doe.virginia.gov/instruction/mathematics/elementary/number_sense_module/nns_gradek.pdf

Literature Connection:

How Many Snails? by Paul Giganti, Jr.
A series of simple questions directs young readers to determine the differences between seemingly similar objects, encouraging them to develop powers of observation, discrimination, and visual analysis. There's plenty of opportunity to practice counting, too (but that's just the beginning!).

What Comes in 2s, 3s, and 4s? by Suzanne Aker
This picture book presents the set number concept through pictures of everyday objects.

Vocabulary Connection for Subitizing and Number Representations

- **Number**: how many of something you have
- **Group**: a collection of items
- **Count**: saying numbers as you touch objects in a group
- **Tally Marks**: lines used to help count
- **Five Frame**: a box used to show five things
- **Ten Frame**: a box used to show 10 things

Discussion

- What other math vocabulary terms do students need to know?

Listening and Speaking Connection

Possible Teacher Questions	Possible Student Responses
How do you know there are four dots on the plate?	I saw 2 on the top and 2 on the bottom.
How did you know there were five dots in the 10 frame?	The top row was full.
How did you make the number 10?	I drew two circles. I filled in the whole 10 frame. I drew 5 orange dots and 5 red dots.

- What other questions could you ask?

Writing Connection: Math Journals

- Using the dot plates you made, show an image, then have students copy it in their math journals.
- After reading the book *Ten Black Dots*, give students some black dot stickers and have them make something and write about it in their journals.
- Tell students to divide their journal page into four sections and draw dots for a given number in different ways: array, line, circle, scattered.
- Tell students to divide their journal paper into four sections and give them a number to represent four different ways: pictures, equation, word, tally marks, etc.

Talk with the people at your table. What is one idea from this morning that you want to use in your classroom? Why?

Closing Activity

Inside/Outside Circle: Participants will divide into two groups. One group will form the inside circle, the other group will form the outside circle. The circle will rotate and participants will share an idea that they have learned, or something they can expand on in their own classrooms.

Key Idea #1

Number sense is a “good intuition about numbers and their relationships. It develops gradually as a result of exploring numbers, visualizing them in a variety of contexts, and relating them in ways that are not limited by traditional algorithms.”

- Hilde Howden

Module 2:

Learning Number Sense through Talking about Math

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Module 2: Learning Number Sense through Talking about Math

Objectives

- Gain an understanding of number sense in the early childhood classroom
- Use number talks to increase number sense
- Learn math instructional strategies and activities to develop number sense using five frames and ten frames

Standards

Count to tell the number of objects

Pre-K Standards	Kindergarten Standards
PK.CC.4. Understand the relationship between numbers and quantities with concrete objects up to 10.	**K.CC.4.** Understand the relationship between numbers and quantities; connect counting to cardinality.
PK.CC.4a. Use one-to-one correspondence to accurately count up to 10 objects in a scattered configuration.	**K.CC.4a.** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object
PK.CC.4b. Understand that the last number name said tells the number of objects counted, up to 10.	**K.CC.4b.** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
PK.CC.5. With guidance and support count to answer "how many?" questions about as many as 10 things arranged in a line or as many as 5 things in a scattered configuration; given a number from 1-10, count out that many objects.	**K.CC.5.** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or any as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

TEAM Alignment

- Teacher Content Knowledge
- Activities and Materials
- Questioning
- Thinking

Reflection

- How do you get your students to think and talk about math?

- What kinds of things do you do in your classrooms that promote mathematical thinking?

Early Math: How Children "Think Math"

How to help children explore and expand math skills

Sarama, Julie & Douglas H. Clements.

Retrieved from http://www.scholastic.com/teachers/article/early-math-how-children-quotthink-mathquot

Jeremy and his classmate, Stacey, were arguing about who had more dessert. "She has more!" declared Jeremy. "I do not!" said Stacey. "We have the same." "No. see, I have one, two, three, four, and you have one, two, three, four, five."

"Jeremy, one of my cookies broke in half. You can't count each half. If you're counting pieces, I could break all yours in half, then you would have way more than me. Put the two halves back together and count. One, two, three, four. Four! We have the same."

Math in the Making

Good early mathematics is broader and deeper than early practice in "school skills." High-quality mathematics is a joy-not a pressure. It emerges from children's play and their natural ability to think. The mathematical thinking that Stacey aptly explained to Jeremy was not only engaging-it also involved a high level of thinking. Stacey had to argue about what the unit was-what it was they were counting-and how the two halves were equivalent to one whole unit. Jeremy also exhibited excellent mathematical thinking skills in knowing that counting was the best way to compare two amounts.

Identify Learning Areas

What mathematics can young children learn? It can be broken down into two main areas: (a) geometric and spatial ideas and (b) numeric and quantitative ideas. Young children possess intuitive and informal capabilities in these areas. Three other mathematical themes that should be woven through experiences in these two main areas are: (a) patterns, (b) sorting and sequencing, and (c) measurement and data.

Plug Math into Routines

Encouraging mathematical development can become part of your everyday routine. Suggest that children count to 15 (slowly!) while they wash their hands before snack. Point out places in their world where numbers are used, read books and sing songs with numbers in them. These are easy activities that don't take long. However, they build the foundation for the type of numerical reasoning that Stacey displayed.

Encourage Geometric Thinking
Children are also naturally interested in shapes and spatial ideas. To encourage geometric thinking and reasoning, play "I Spy" using shape descriptions. Ask children what shapes they see in the classroom.

Make the Connections
Ask children questions, such as, "What number do you think will be on the next page? How do you know?" This encourages children to see counting as a pattern, a pattern through which they have the ability to predict what comes next. It also connects patterning to numbers. Likewise, when children naturally talk about relative size ("You have the biggest shovel"), encouraging them to measure to "prove" their prediction connects numbers and geometry.
Mathematical thinking comes naturally to young children and can develop substantially during the early years.

Reflection

1. What do you think the author means by “good early mathematics is broader and deeper than early practice in “school skills”?”

2. Describe the mathematical thinking that Stacey and Jeremy used in their conversation.

3. What routines are suggested to help build a foundation for numerical reasoning?

Group Discussion

Talk with the people at your table about your answers. Using chart paper, generate a list of routines you use to develop mathematical thinking for number sense with your students.

Key Idea #2

Incorporating math into all content areas and daily routines promotes mathematical thinking by students.

Number Talk Routines in the Early Childhood Classroom

What is a Number Talk?

- A Number Talk is a short, ongoing daily routine that provides students with meaningful ongoing practice with computation. A Number Talk is a powerful tool for helping students develop computational fluency because it prompts students to use number relationships and the structures of numbers to add, subtract, multiply, and divide.
- Number Talks should be structured as short sessions alongside (but not necessarily directly related to) the ongoing math curriculum. Math Talks are not intended to replace current curriculum or take up the majority of the time spent on mathematics. Teachers should spend only 5-10 minutes on Number Talks.
- Number Talks are most effective when done every day.
- The purpose of a Number Talk is to help students develop computational fluency as well as number sense.
- During a Number Talk, the teacher only serves as a recorder and facilitator without teaching new content.

What can math talk look like in pre-K?

- “Preschool teachers who use numbers in their everyday speech may aid their students' math abilities, according to new research published in January's *Developmental Psychology* (Vol. 42, No. 1). Even seemingly trivial instances of "math talk," such as saying "You two get your coats," instead of "You guys get your coats," may be related to improvement in four- and five-year-olds' math skills, says study author Raquel Klibanoff, PhD, who conducted the research as a postdoctoral fellow at the University of Chicago.”
- “Those students who were in classrooms where the teachers used many instances of math talk tended to improve more over the course of the school year than students who were less exposed to math vocabulary, the researchers found. What's more, the improvements were unrelated to general teacher quality, the complexity of the teachers' sentence structure or student socioeconomic status.”

Retrieved from http://www.apa.org/monitor/feb06/math.aspx

Conducting a Number Talk

- In Kindergarten, use pictures or models such as 10 frames. Your questions can simply be "How many?" In pre-K, teachers can incorporate math talk into everyday routines such as, "You two go get water" instead of being more general.
- Students should also be given the option of using manipulatives to solve the problems.
- Ask for four to five answers and write them down whether they are right or wrong. When you first start, you may consider just having students share answers without writing them down, especially in Kindergarten.
- Ask for explanations for the process of solving and write them down. Write exactly what the student says and ask for clarification. In the beginning, especially in Kindergarten, you may not want to write everything down, but rather focus on the "math discussion." Be sure to repeat back what the child has said so he/she feels heard.
- Ask students what mistakes they made.
- Give the same response to each student whether they are right or wrong. Give the same response for each strategy. You do not want students coming up with "crazy" ways to solve a problem just to impress you. You also don't want students to worry about being wrong.

- Retrieved from http://www.cobbk12.org/bullard/numbertalksk-2.pdf and based on *Number Talks* by Sherry Parrish, Math Solutions 2010.

Number Talk Video

The following video shows a teacher doing a number talk with her class. Focus on the teacher and what she does on the video.

https://www.youtube.com/watch?v=62epCIFdRa0

What did the teacher do in the video to help develop number sense?

This next video is another teacher doing a very similar number talk. This time, focus on what the students say and how they show their thinking.

https://www.youtube.com/watch?v=8D-qejdIlFg

What were some math strategies the students used in identifying the number sets?

Key Idea #3

When using Number Talks, the teacher should shift from thinking "What answer did you get?" to "How did you get that answer?"

Group Activity

With the people at your table, decide who will be the "teacher" and conduct a number talk. The rest of you will be the "students." Using the dot plates you made this morning, practice doing a number talk. Use the table below as a guide:

Teacher's Role	Students' Roles
Give students a signal to use to show when they have an answer ready to share.	Hold fist on chest, thumb up when they know the answer
Hold up a dot card for a few seconds and ask "How many do you see? Give some think time.	Give the answer when called on
Call on several students to give their answer	Explain how they saw the amount
Restate a student's strategy for telling how many they saw	

Talk with the people at your table about number talks. What are some tips you can share? What suggestions can you make?

Number Talk Resources

http://thekindergartenconnection.com/number-talks-in-kindergarten/ (blog that shows a great number talk routine)

http://bpsassets.weebly.com/uploads/9/9/3/2/9932784/number_talks_kindergarten_resource.pdf (number talk information and dot cards that can be used)

Using Five Frames and Ten Frames

What are Five and Ten Frames?

Five and ten frames are equal-sized rectangular boxes in a row where each box is large enough to hold a counter. The five frame is arranged in a one-by-five array. A ten frame is a set of two five frames or a two-by-five array.

Five frame:

Ten frame:

How do Five and Ten Frames help students?

Five and ten frames help students to relate given numbers to five and 10 by providing a visual image. The frames may be filled in from left to right so that students can learn to *subitize.* Using five and ten frames encourages counting strategies beyond counting by one. Students learn to think about combinations of numbers that make other numbers, e.g., seven is two more than five, or nine is one less than 10. These number relationships help build the foundation for the development of more complex mental computations. Students start with the five frames before moving to ten frames and may explore double ten frames later to develop a better understanding of place value.

What materials are needed to use frames?

It is recommended that every child have a five frame to begin and, when developmentally ready, they should also have a ten frame. Blackline masters of frames can be mounted on cardboard. Students also need counters (at least 10 per child) to place in and beside the frames for counting.

Classroom Activities

- Give each student a five frame. Orally give students a number to fill in on their five frame, explaining to them to only put one counter in each space. Ask them to explain ways they have displayed the quantity.
- Once the students have displayed a number, ask "How many more counters are needed to make five?" This reference to five build student thinking and understanding of what five is as an anchor number.
- Call out numbers greater than five and have students place those additional counters outside the frame so they see that seven is two more than 5.
- Once students have had experience with five frames, repeat the above activity with the ten frame cards.
    - **Note:** When using the ten frames, ask students to fill the top row up first, before moving on to the second row, as this will provide a "standard" way to show numbers and reinforce the concept of fives and tens as anchors.
- Using two-sided counters, find all the ways to make five (or 10).
- When students have had experience with five or ten frames play a game by quickly flashing a filled frame and ask how many dots there were. Encourage students to share strategies of how they could tell without counting. This is an example of how five and ten frames can be used in Number Talks.
- Call out numbers as a shared class experience and students build that number on their frames with two-color counters. Ask different students to share how they built their number.
- Hold up a frame with some boxes already filled and say "I wish I had five (or 10)." Students figure out how many more counters are needed to make that number.
- One student arranges counters on the ten frame and hides it from a partner. The partner can ask "Yes or No" questions to figure out the hidden number. Is the top row full? Are there more than three spaces empty?
- Dot Card/Ten Frame Match --Students match a dot card to a ten frame with the same amount.

Adapted from *Teaching Student-Centered Mathematics: Volume One, Grades, K-3*. John Van de Walle, Boston: Pearson, 2006. ISBN 205-40843-6 Retrieved from http://www.edugains.ca/resources/LearningMaterials/ManipulativesSupport/TipSheets/Manipulatives_Frames-five-ten.pdf

Five/Ten Frame Resources

Interactive resources

http://illuminations.nctm.org/activitydetail.aspx?id=74

http://illuminations.nctm.org/activitydetail.aspx?ID=75

https://www.youtube.com/watch?v=t8U_zZ-rW1E ten frame flash video

Classroom lesson ideas

http://www.k-5mathteachingresources.com/ten-frames.html

http://www.k-5mathteachingresources.com/kindergarten-math-activities.html

http://mathplc.com/sites/default/files/DotCardTenFrameActivities.pdf

Make and Take Activity

Using the dot stickers and five/ten frames, create some frames with different combinations of numbers. Talk with the people at your table about ways you use five and ten frames in your classroom. Make a list here, and be ready to share some of your ideas with the room.

Reflection

Ways I can use Five/Ten Frames in my classroom:

Key Idea #4

Five frames and ten frames are one of the most important models to help students anchor to five and 10.

Retrieved from http://mathplc.com/sites/default/files/DotCardTenFrameActivities.pdf

Literature Connection

A great way to tie in literacy with math instruction is using books to practice number sense. Read the book through the first time so the students are familiar with the story. Re-read it and let them practice with the frames and manipulatives to make the amounts in the story. You can also read to a particular page in the book, and then do a number talk about what they see. You can have the student's make predictions about what number will come next in the story.

My Little Sister Ate One Hare by Bill Grossman

In this funny counting book, a hungry little sister eats everything from one hare to 10 peas. Great for using ten frames while reading.

Ten Apples Up on Top by Theo LeSieg

A lion, dog, and tiger try to do tricks while counting the apples on their heads.

Vocabulary Connection for Five and Ten Frames

- **More**: a bigger group of something
- **Less**: a smaller group of something
- **Five Frame**: a box used to show five things
- **Ten Frame**: a box used to show 10 things

Can you think of other math vocabulary that students need to know?

Listening and Speaking Connection

How can you incorporate listening and speaking skills into the activities shared today? What questions can you ask students? What type of responses should you hear?

Possible Teacher questions	Possible Student Responses
How did you know how many more dots you needed to make 10?	I counted six and saw four boxes were empty, so I needed four more to make 10.
How did you know there were six dots on the 10 frame so quickly?	I saw that the top row was full and there was one more on the bottom row.
What other ways could you make eight on the 10 frame?	I could put five red circles and three blue circles. I put four on the top row and four on the bottom row.

What other questions could you ask?

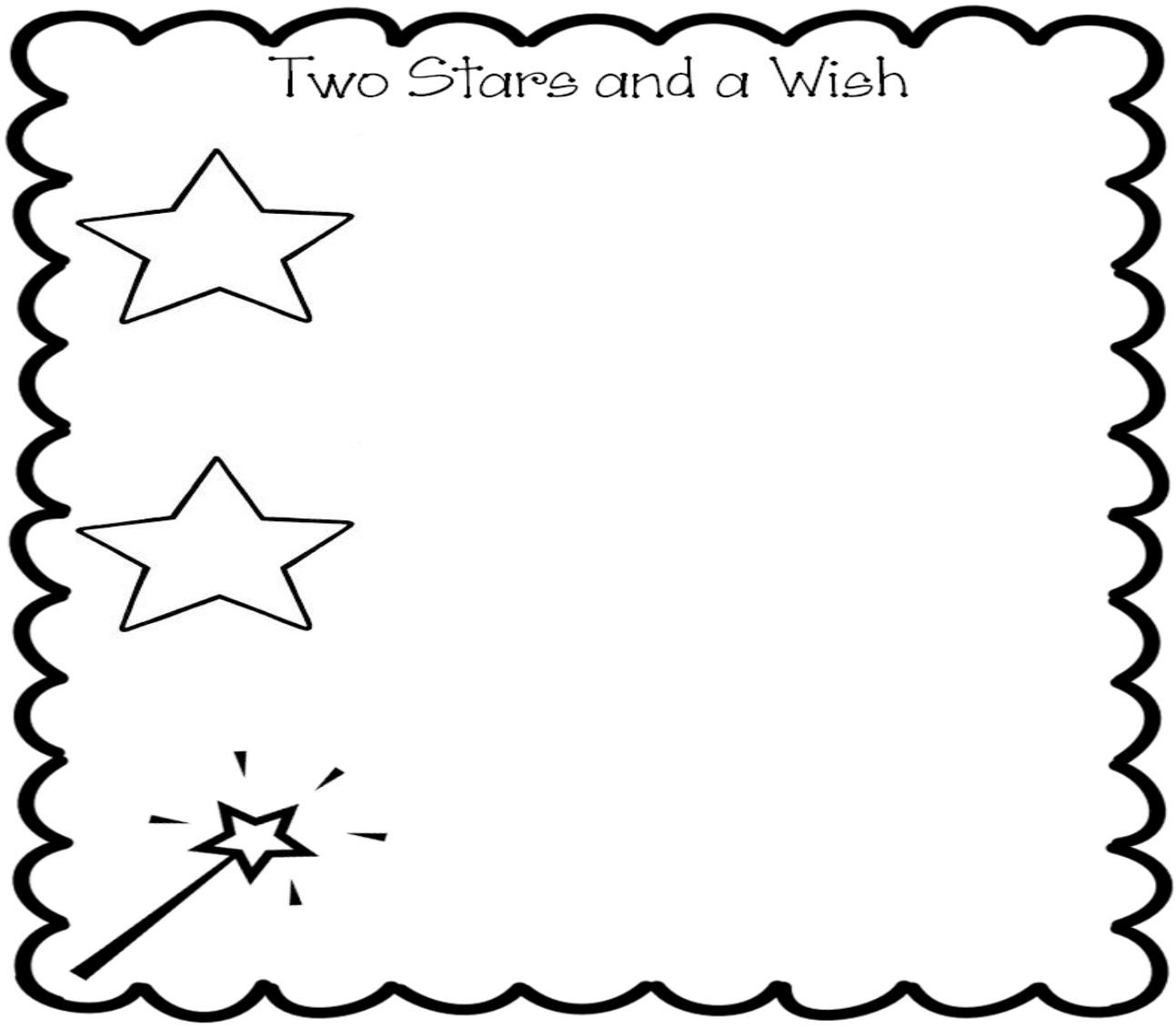
Writing Connection: Math Journals

- Using the 10 frames you made, play Ten Frame Flash, and then have students copy it in their math journals.
- After reading the book, *Ten Apples Up on Top* have students draw themselves and write about how many apples they could hold on their head. Give them red stickers (apples) to show the number.
- Tell students to divide their journal paper into 4 sections and give them a number to represent 4 different ways on a 10 frame. (you can buy 10 frame stamps from Oriental Trading)

Talk with the people at your table. What is one idea from this morning that you want to use in your classroom? Why?

Closing Activity: Two Stars and a Wish

Think about all we have learned today. In the space provided, please list two things that you "shine" at in your classroom. These may be ways your classroom is rich in numeracy, activities you do to encourage mathematical thinking, or books you read that help children with number sense. List these beside the stars. Then, think of one "wish" you have to improve upon in your teaching. It may be something new you want to try, or something you already do.



Retrieved from http://lookingfromthirdtofourth.blogspot.com/2015/02/learning-goals-achievement-levels-and-2.html

Module 3:

Teaching Addition and Subtraction

[TAB PAGE]

Module 3: Teaching Addition and Subtraction

Objectives

- Gain an understanding of the CRA model
- Learn to use the Think Aloud strategy for math
- Learn math instructional strategies and activities to teach addition and subtraction

Standards

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Pre-K Standards	Kindergarten Standards
PK.OA.1. Represent real-world addition (putting together), and subtraction (taking from) problems up through five with concrete objects or by acting out situations.	**K.OA.1.** Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
PK.OA.2. Solve addition and subtraction problems using objects for problems up through five.	**K.OA.2.** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
PK.OA.3. Compose and decompose numbers to five by using objects or drawings (may be an extension activity after reading a book).	**K.OA.3.** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing

TEAM Alignment

- Teacher Content Knowledge
- Activities and Materials
- Questioning
- Thinking

Reflection

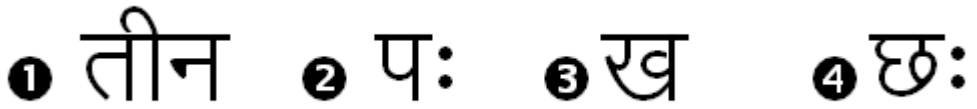
How do you teach addition and subtraction to your class? What do you do first? What steps do you take to ensure all students learn the concept? List your thoughts below:

Group Sharing

Share with your table the steps you would take to teach the lesson. Are your ideas similar to the other participants?

The CRA Method of Instruction

Look at the symbols below. Circle the two symbols that are numbers.



How did you do? Did you see any distinguishing characteristics that helped you make your determination?

The symbols look similar. What does this have to do with math? Until a child knows that five means 5 fingers, or 5 dots, or 5 toy cars, or 5 years old, the symbol doesn't look much different than a letter. It has some curved lines and some straight lines, like the letters *a, b,* or *p.*

Twenty years ago, Singapore occupied the space squarely at the bottom of the world rankings in math. Now, they are among the top. What changed? They adopted a teaching philosophy that is built on the concrete, representational, abstract (CRA) sequence of instruction. They call it CPA, with the P standing for pictorial. Regardless of the letters used, this sequence of instruction is based on the research of Jerome Bruner. It says that students must experience and interact with a concept to develop a true understanding.

Retrieved from http://www.mathcoachscorner.com/2015/05/why-cra/

CRA Video

https://www.youtube.com/watch?v=weCPBlJVSrI

Concrete-Representational-Abstract Instructional Approach

What is the Concrete-Representational-Abstract (CRA) Instructional Approach?
The CRA Instructional Approach is "an intervention for mathematics instruction that research suggests can enhance the mathematics performance of students" (Hauser). The approach is a "three-part instructional strategy, with each part building on the previous instruction to promote student learning and retention and to address conceptual knowledge" (Hauser). The three parts are as follows:

- **Concrete:** In this stage, the teacher begins instruction by modeling each mathematical concept with concrete materials. In other words, this stage is the "doing" stage, using concrete objects to model problems.
- **Representational:** In this stage, the teacher transforms the concrete model into a representational (semi-concrete) level, which may involve drawing pictures; using circles, dots, and tallies; or using stamps to imprint pictures for counting. In other words, this is the "seeing" stage, using representations of the objects to model problems.
- **Abstract:** In this stage, the teacher models the mathematics concept at a symbolic level, using only numbers, notation, and mathematical symbols to represent the number of circles or groups of circles. The teacher uses operation symbols (+, –, x, /) to indicate addition, multiplication, or division. This is the "symbolic" stage, where students are able to use abstract symbols to model problems (Hauser).

In the classroom, this approach is a facilitating framework for students to create meaningful connections between concrete, representational, and abstract levels of thinking and understanding. Students' learning starts out with visual, tangible, and kinesthetic experiences to establish basic understanding, and then students are able to extend their knowledge through pictorial representations (drawings, diagrams, or sketches) and then finally are able to move to the abstract level of thinking, where students are exclusively using mathematical symbols to represent and model problems (Hauser).

Studies have shown that "students who use concrete materials develop more precise and more comprehensive mental representations, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations" (Hauser).

What is the Purpose of the CRA Approach?

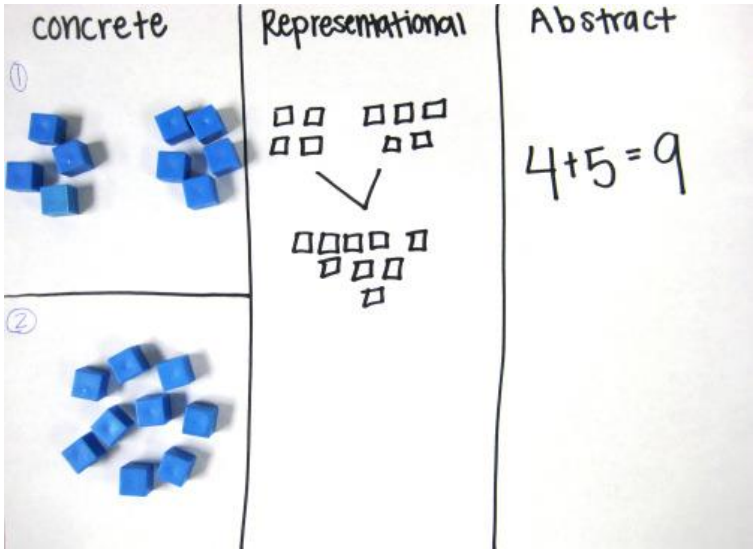
The overarching purpose of the CRA instructional approach is to "ensure students develop a tangible understanding of the math concepts/skills they learn" (Special Connections, 2005). Using their concrete level of understanding of mathematics concepts and skills, students are able to later use this foundation and add/link their conceptual understanding to abstract problems and learning. Having students go through these three steps provides students with a deeper understanding of mathematical concepts and ideas and provides an excellent foundational strategy for problem solving in other areas in the future (Special Connections, 2005).

How Can I Implement the CRA Approach in My Classroom?

One of the first and most important steps to implementing the CRA approach in the classroom is to "use appropriate concrete objects to teach particular math concepts/skills. Discrete materials (e.g. counting objects such as beans, chips, unifix cubes, popsicle sticks, etc.) are especially helpful since students can see and feel the attributes of the objects they are using" (Special Connections, 2005).

Once students have mastered the concrete level of performance, introduce appropriate drawing procedures where students problem solve through drawing simple representations of the concrete objects they previously used (e.g. tallies, dots, and circles). "By replicating the movements students previously used with concrete materials, drawing simple representations of those objects supports students' evolving abstract understanding of the concept/skill" (Special Connections, 2005).

Finally, after a student demonstrates a thorough understanding of the representative level, use appropriate strategies to help students move from that representational level to the more abstract level. If students have trouble moving to the abstract, "re-teach the mathematics concept/skill using appropriate concrete materials and then explicitly show the relationship between the concrete materials and the abstract representation of the materials" (Special Connections, 2005). If students already have a concrete level of understanding of that concept/skill, "provide students opportunities to use their language to describe their solutions and their understandings of the mathematics concept/skill they are learning" (Special Connections, 2005).



Retrieved from https://mathteachingstrategies.wordpress.com/2008/11/24/concrete-and-abstract-representations-using-mathematical-tools/

Key Idea #5

By linking learning experiences from concrete to representational to abstract levels of understanding, the teacher provides a graduated framework for students to make meaningful connections.

- Retrieved from http://fcit.usf.edu/mathvids/strategies/cra.html

Group Activity:

With the people at your table, plan a lesson using addition or subtraction based on the CRA approach. Use ideas from the lists below, or come up with your own. List the steps you would take in teaching the lesson on the chart paper provided.

Concrete Manipulative Examples:

- colored chips
- beans
- unifix cubes
- candy (ex. Skittles)
- popsicle sticks
- bears

Representational Examples:

- tally marks
- dots
- circles
- pictures of objects

Abstract Examples:

- numbers
- equations
- worksheets
- flash cards
- word problems

- Retrieved from https://makingeducationfun.wordpress.com/2012/04/29/concrete-representational-abstract-cra/

Using Think Alouds to Teach Addition and Subtraction

Jigsaw Activity

Participants will number off one through five. The same number groups will move together and read their section of the article below. After discussing the main points, participants will move back to their original table and "teach" their part of the article.

Group 1: Talk moves that engage students in discourse
Group 2: The art of questioning
Group 3: Using student thinking to propel discussions
Group 4: Setting up a supportive environment
Group 5: Orchestrating the discourse

How to Get Students Talking!
Generating Math Talk That Supports Math Learning
De Garcia, Lisa Ann.

Due to the attention in the last few years on discourse and its importance to student learning, educators nationwide are finding that they can help children become confident problem solvers by focusing on getting them to talk and communicate in partnerships, small groups, whole groups, and in writing. In addition, English Language Learners are flourishing as they experience focused opportunities for talking and trying on new mathematical vocabulary.

So what exactly is discourse? What are the teaching practices associated with successfully establishing an environment to support it, and as a result, to improve mathematical proficiency? How does one begin to elicit meaningful talk during math lessons? As a profession, we share a vision about the role student discourse has in the development of students' mathematical understanding, but are often slow to bring the students along. Children do not naturally engage in this level of talk.

This article addresses the above questions and concerns—and more. It opens with a look at discourse through NCTM's definition and its involvement with the Common Core State Standards. It then focuses on literature available on discourse, specifically the book *Classroom Discussions,* and addresses five teaching practices focused on the *how to* of getting students talking about mathematics. The article concludes with journaling insights on discourse from a

kindergarten and second-grade classroom. This article is by no means an exhaustive list of discourse "to dos;" hopefully it will however get us all started in thinking about and implementing best talk practices.

What is Discourse in the Mathematics Classroom?

NCTM's Definition

The National Council of Teachers of Mathematics (NCTM) in their 1991 professional standards describes discourse as ways of representing, thinking, talking, agreeing, and disagreeing; the way ideas are exchanged and what the ideas entail; and as being shaped by the tasks in which students engage as well as by the nature of the learning environment.

Learning from Literature on Discourse

One of the leading resources for discourse is *Classroom Discussions: Using Math Talk to Help Students Learn* (Chapin, O'Connor, and Anderson 2009). This resource and others highlight five teaching practices associated with improving the quality of discourse in the classroom.

Practice 1: Talk Moves That Engage Students in Discourse

For the first practice, the authors of *Classroom Discussions* propose five productive talk moves that can get talk going in an otherwise silent classroom:

- revoicing (by the teacher)
- restating someone else's reasoning (by the student)
- applying their own reasoning to someone else's (by the student)
- prompting for further participation (by the teacher)
- wait time (by the teacher)

The first is ***revoicing***. An example would be, "So you are saying that…" This ***revoicing*** allows the teacher to check in with a student about whether what the student said was correctly heard and interpreted by the teacher or another student. A way to encourage students to listen to their peers is through asking them to ***restate someone else's reasoning***, such as, "Can you repeat what he just said in your own words?" Another way is to ask students to ***apply their own reasoning to someone else's*** using questions such as "What do you think about that?" and "Do you agree or disagree? Why?" This helps prevent students from just thinking about what they want to share and focuses their attention on what their classmates are saying. It also helps to strengthen the connections between ideas.

Simple questions such as, "Would someone like to add on?" are ways teachers can ***prompt for further participation***. This helps elicit more discussion when not many students are talking, especially when they are not accustomed to explaining their thinking. Again it helps students to

tune in to what others are saying so that they are able to expand on someone else's idea. Perhaps the most valuable talk move suggested by Chapin, O'Connor, and Anderson is the use of ***wait time***. Often teachers are too quick to answer their own questions when no one chimes in. Children quickly become accustomed to this. Waiting provides think time and sets the expectation that someone will indeed respond and that the teacher will wait until someone does. Another important use for wait time is to provide English Language Learners, or anyone who needs extra time, with an opportunity to process the question and formulate an answer. One teacher reported that in his initial uses of wait time, one of his English Language Learners was able to participate in class discussion for the first time.

Practice 2: The Art of Questioning

Questioning is another critical component in supporting students to engage in meaningful discussions. The NCTM Standards outline roles questions have in the math classroom. The first role, ***helping students to work together to make sense of mathematics***, is addressed by the five talk moves discussed above. The second role, ***helping students to rely more on themselves to determine whether something is mathematically correct,*** can be supported by questions such as:

- How did you reach that conclusion?
- Does that make sense?
- Can you make a model and show that?
- Does that always work?
- Is that true for all cases?

Questions designed to ***help students to learn to reason mathematically*** would include:

- Can you think of a counter example?
- How could you prove that?

To ***help students to learn to conjecture, invent, and solve problems***, the teacher might ask:

- What would happen if?
- Do you see a pattern?
- Can you predict the next one?
- What about the last one?

Finally, teachers use questions to ***help students connect mathematics, its ideas and applications*** by asking:

- How does this relate to…
- What ideas that we have learned were useful in solving this problem?

Practice 3: Using Student Thinking to Propel Discussions

Because discussions help students to summarize and synthesize the mathematics they are learning, the use of student thinking is a critical element of mathematical discourse. When teachers help students build on their thinking through talk, misconceptions are made clearer to both teacher and student, and at the same time, conceptual and procedural knowledge deepens. When doing so, the teacher must be an active listener so she can make decisions that will facilitate that talk. She also needs to respond neutrally to errors, so that the students can figure out misconceptions themselves.

For example, the teacher can ask the whole class, "What do you think about that?" when a student offers an incorrect strategy or can ask the rest of the class to prove whether or not the strategy works. Through the conversation, the misconception becomes apparent to the class. This practice results in an authentic discussion focused on the mathematics and not on the individual student. The teacher also needs to be strategic about who shares during the discussion, since it is not a show-and-tell session, and choose ideas, strategies, and representations in a purposeful way that enhances the quality of the discussion.

Practice 4: Setting Up a Supportive Environment

When setting up a discourse-rich environment and one that enhances student engagement, both the physical and emotional environment must be considered. Teachers who have studied engagement find that it is very effective if students face each other, either sitting in a circle or semi-circle on the floor or sitting in chairs arranged in a circle. Teachers can sit with students as part of the circle to encourage peer-to-peer discussion.

If teachers are still having difficulty getting children to talk, they can remove themselves from the group and stand outside the circle. As a result, students are left looking only at each other, which encourages them to direct their comments to one another.

Careful consideration of the placement of visual aids and mathematically related vocabulary is important in supporting the level of talk. If charts are not visually accessible when they need to be, they will likely not be resourced by the students during whole group conversations. To increase the extent to which English Language Learners participate in group discussions, having related vocabulary and sentence frames where they can be easily accessed is critical.

For rich discussions, the emotional environment of the classroom must be safe and must be one where students want to learn and think deeply about the mathematics. When these elements are not present, the discussion stays at the surface level. Imagine a third grade classroom where the teacher introduces division for the first time and is met with cheers. It can happen! It happens when the value is on learning, challenging each other, and working together

to solve problems as opposed to just getting the right answer. For more on setting up a supportive classroom environment for discourse, see Chapter 8 of *Classroom Discussions.*

Practice 5: Orchestrating the Discourse

The teacher becomes not unlike a conductor as he supports students to deepen their understanding of mathematics through a carefully orchestrated environment. In *Orchestrating Discussions,* Smith, Hughes, Engle, and Stein outline the *Five Practices Model*, which gives teachers influence over what is likely to happen in a discussion.

The Five Practices Model

The teacher's role is to:

- Anticipate student responses to challenging mathematical tasks.
- Monitor students' work on and engagement with the tasks.
- Select particular students to present their mathematical work.
- Sequence the student responses that will be displayed in specific order.
- Connect different students' responses and connect the responses to key mathematical ideas.

Even if the teacher is focused, students still need to be held accountable. Otherwise, the discussion will be unproductive. A lot of explicit teaching must go into how to engage in each level of discussion: whole group, small group, and partnerships. In the younger grades, one will find teachers showing students exactly what they should look like and sound like when discussing their thinking. Teachers may say things like, "Today in math, we are going to practice turning and talking with our partner. When I say go, you are going to turn like this and look at your partner. When I say stop, you are going to turn around and face me. Let's practice that right now."

Even older students need to be explicitly taught what to do and say. A teacher might teach how a partnership functions by saying, "It sounds like you have an idea and you have an idea, but what seems to be lacking is for you two to put your ideas together to come up with a solution. So, what is your plan?" One very effective method of holding students accountable is to let them know exactly what they should be saying when they are talking in their partnerships or small groups. For example, "Today, when you are talking to your partners and describing your solid shapes, I expect to hear you using the words faces, edges, and vertices." It is also supportive to let students know what they should be focusing on when someone is sharing a strategy, so they have a lens for listening, which heightens the level of engagement. A teacher might say, "When

he is sharing his thinking, I want you to be thinking of how his way is similar or different to your way." Students need to be aware of themselves as learners, and a great way to heighten this awareness is through self-evaluation and goal setting. Sometimes the child is the last one to know that he is distracting or not listening. Part of developing a safe culture is supporting students in being open with each other regarding their strengths and weaknesses so they can improve their communication skills and behaviors.

It is wonderful to hear one child compliment another when she has participated for the first time or give gentle correction when another has been dominating the conversation. This level of self-awareness happens through consistent venues such as class meetings and tracking the progress of personal goals related to participation in mathematical discussions. The more students open up about themselves as learners, the deeper the relationships and, as a result, the deeper the trust.

Five Teaching Practices for Improving the Quality of Discourse in Mathematics Classrooms

1. Talk moves that engage students in discourse
2. The art of questioning
3. Using student thinking to propel discussions
4. Setting up a supportive environment
5. Orchestrating the discourse

- Retrieved from http://www.mathsolutions.com/documents/how_to_get_students_talking.pdf

After sharing your portion of the article with the other participants at your table, talk about how these strategies would work in your classroom. What listening and speaking skills would you incorporate into the Think Aloud?

Practicing Discourse with Think Alouds

The "think-aloud" process is usually introduced in four steps, gradually transferring responsibility to students:

1. The teacher reads a problem and stops as needed to explain her thoughts. Students listen. They all solve the problem together.
2. The teacher reads the problem and stops often. Students express their thoughts at each point (and often write them). The whole class, led by the teacher, solves the problem together.
3. The teacher reads the problem, allowing students to signal stopping points as thoughts occur to them. Students solve the problem individually, and then discuss their interpretations of it and solution strategies.
4. Students do this together, in pairs. They work together to solve the problem.

A benefit of this strategy for students who need additional support is that they also listen to the thinking of their classmates, enabling them to learn additional strategies from each other.

- Retrieved from http://math-problem-solving.wiki.inghamisd.org/Think-Aloud

Think Aloud video example

http://www.showme.com/sh/?h=1U4ux4S

Key Idea #6

"Thinking aloud" requires students to talk through the details of the problem, the decisions they have made as they try to solve the problem, and the reasoning behind those decisions.

- Retrieved from http://www.ldonline.org/article/63842/

Practice a “Think Aloud”

At your table, choose someone to be the teacher and practice a think aloud. Decide together on a problem that you would use with your students. How would this look different in pre-K and K? Would you use manipulatives in your Think Aloud? Why or why not?

Addition and Subtraction Games and Activities

Carousel Activity

Number off one through eight. Each number group will stand by their number chart to begin the activity. The questions are the same for addition and subtraction, with four charts for each operation. On the chart, list any ideas that you have about teaching addition and subtraction.

Chart Headings:

- What games and activities do you use to develop student's conceptual understanding?
- What concrete, representational, or reasoning strategies do you teach your students?
- What books do you use to teach addition or subtraction?
- What online resources do you use?

Common Addition Strategies

- Draw a picture
- Counting on
- Using doubles
- Fact Families (flip flop the equation)

Addition Activities in the Classroom

Guess My Number

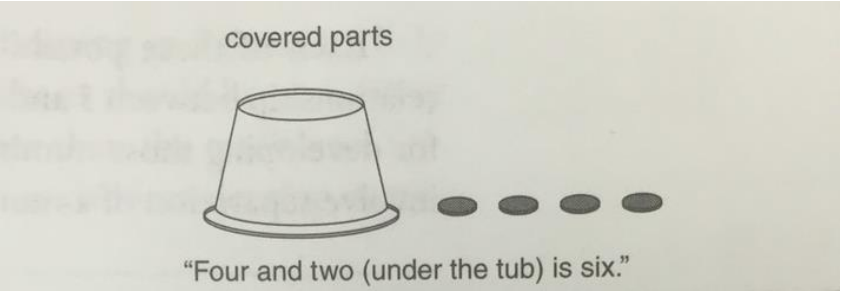
3	8
6	10

- ✓ I can use the doubles strategy to get my number.
- ✓ My number is less than seven.
- ✓ My number is 3 + 3.

1. Draw a table with numbers like the one above.
2. Give students clues, one by one so they can guess your number.
3. Use this for addition or subtraction.

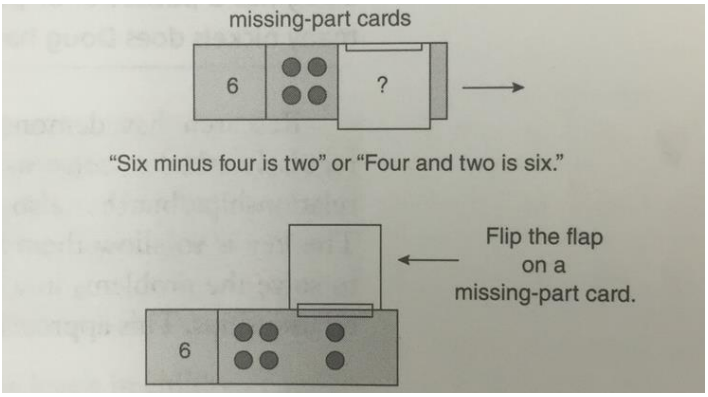
Covered Part

1. Choose a target amount.
2. A set of counters equal to the target amount is counted out, and the rest are put aside.
3. One child places the counters under a solid cup and then pulls some out into view. (This amount can be none, all or any amount in between.)
4. The partner says the two parts that makes the whole. For example, "Four and two is six"
5. If there is a hesitation or if the hidden part is unknown, the hidden part is immediately shown.



Missing Part Cards

1. For each number from four to 10, make missing part cards using strips of cardstock measuring 3 X 9 inches (this will ensure that a post it will fit over it.)
2. Each card has a numeral for the whole and two dot sets with one set covered by a post it.
3. Children use the cards by saying the two parts that make a whole such as "four and two is six."



Make and Take—Missing Part Cards

- Take five sentence strips and cut them in half.
- Use the markers and stickers to make missing part cards.
- Choose numbers that are appropriate for your classroom.

Literature Connection for Addition

Quack and Count by Keith Baker
There are seven ducklings on a page. The book shows all the ways to add numbers to make seven with a rhyming text. This book would be great to use for a **number talk** about addition. Cover the text and show the picture. Have students tell how many ducklings they see and how they see them.

Animals on Board by Stuart J. Murphy
This is a Math Start book. The illustrations show big trucks carrying animals. One truck has some animals, and then another truck comes along with some more of the same animals. The book shows the equation, but you could cover it up and ask kids how many animals are all together. You could read the book without showing the pictures at first and use the **think aloud strategy**.

Common Subtraction Strategies

- Counting Back
- Related Facts (fact families)
- The Zero Rule

Subtraction Activities in the Classroom

Songs and Finger Plays that Teach Subtraction:

- Five Little Monkeys
- Five Little Ducks
- Five Little Speckled Frogs
- There Were Ten in the Bed

Games that Teach Subtraction

Bowling

1. Set up 10 cups in a bowling pattern.
2. The first player rolls the ball and counts how many pins were knocked over and records that number on their recording sheet.
3. Using a subtraction strategy, they find out how many pins are left.
4. The student can check their answer by counting how many pins were not knocked down.
5. Now it's player two's turn!

Musical Chairs

After reading the book *Monster Musical Chairs*, set up chairs and play!
Let one group of students play the game, while the other students record the equations, and then switch.

Literature Connection for Subtraction

Monster Musical Chairs by Stuart J. Murphy
This is another Math Start book. Ten monsters are playing musical chairs and each time the music stops, one goes away. It is a great way to introduce the concept of subtraction by one.

Five Little Monkeys Sitting in the Tree by Eileen Christelow
This book is a favorite rhyming book to use with subtraction. It is fun to act out the story with the students being the monkeys.

Vocabulary Connection for Addition and Subtraction

- **Add**: putting two or more groups together
- **Subtract**: taking some away
- **Plus**: a sign that means "to add groups together"
- **Minus**: a sign that means "to take away"
- **Equals**: the total after you add or subtract
- **Sum**: how many you have in all
- **Difference**: how many you have left
- **Equation**: the number sentence

What other math vocabulary do students need to know for addition and subtraction?

Listening and Speaking Connection

How can you incorporate listening and speaking skills into the activities shared today? What questions can you ask students? What type of responses should you hear?

Possible teacher questions	Possible student responses
Can you solve this equation? How did you get the answer?	”3 + 2 = 5” I had three fingers on one hand, and two fingers on the other hand. I counted them all together.
What equations can you think of that equal 10?	Students respond by giving equations and sharing their thinking, either with addition or subtraction
I have four cookies, and I want two more. How many will I have all together?	Students respond by counting out four cubes, then adding two more, talking through their thinking as they solve the problem.

What other questions could you ask?

Writing Connection: Math Journals

- Print story problems on labels and place in student math journals. Students draw pictures and write equations to show the answer.
- Have students make up their own story problems and write them in their journals.
- Give students a prompt: How many pets do you have all together?
    - Let students work in pairs. One student writes a sentence to begin an addition story, and the partner writes the next sentence. They solve the problem together. For example: Student 1 writes: I have three cats. Student 2 writes: I have two dogs. Both students write: We have five pets.
    - Instead of writing sentences, students can also draw pictures.

Talk with the people at your table. What is one idea from this morning that you want to use in your classroom? Why?

Closing Activity

What "stuck" with you today? Write your answer on a sticky note and place it on the chart as you leave for lunch.

Module 4:

Teaching Numbers in Base 10

[TAB PAGE]

Module 4: Numbers in Base 10

Objectives

- Gain an understanding of the learning progression for numbers in base ten
- Learn math instructional strategies and activities to teach numbers in base ten

Standards

Work with numbers 11–19 to gain foundations for place value.

K.NBT.1 Compose and decompose numbers from 11 to 19 into 10 ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of 10 ones and one, two, three, four, five, six, seven, eight, or nine ones.

TEAM Alignment

- Teacher Content Knowledge
- Activities and Materials
- Questioning
- Thinking

Reflection

Think about all of the mathematical ideas we have talked about up to this point. How would you use any or all of these ideas to help your students learn about numbers in base ten?

Learning Progressions in Math

Children follow natural developmental progressions in learning and development. As a simple example, children first learn to crawl, which is followed by walking, running, skipping, and jumping with increased speed and dexterity. Similarly, they follow natural developmental progressions in learning math; they learn mathematical ideas and skills in their own way. When educators understand these developmental progressions, and sequence activities based on them, they can build mathematically enriched learning environments that are developmentally appropriate and effective.

- Retrieved from http://www.child-encyclopedia.com/numeracy/according-experts/learning-trajectories-early-mathematics-sequences-acquisition-and

The table below outlines what we want students to be able to understand and do when learning numbers in base ten.

What skills do your students need to have mastered to begin on this learning progression? Talk with your group, and, in your opinion, list the skills that a child would need to have mastered before beginning work on numbers in base ten.

Understand	Do
Concepts/Big Ideas • Quantities represented by numbers can be composed and decomposed into part-whole relationships (by place value up to 20 in K). • The base 10 number system allows for a new place-value unit by grouping ten of the previous place-value units (and this process can be iterated to obtain larger and larger place-value units). • The value of a digit in a written numeral depends on its place, or position, in a number. • Each composition or decomposition can be recorded by a drawing ***or*** equation (e.g., 18 = 10+ 8). **Models/Representations** • Discrete counters • Counters that can stack, group together (tiles, straws banded together, snap cubes, e.g.) • Rekenreks • Five frames, 10 frames • Tallies to five, 10	**K.NBT.1 Gain foundations for place value** • Represent numbers 11-19 with ones units • Associate the number 10 with a collection of ten ones (a "tens" unit) • Representing 11-19: compose ten ones (units) into a bundle of 10 to enable students to "see" a grouping of ten represented • Draw pictures representing the numbers 11-19 showing a bundle/group of ten with further units to represent the ones/units **Compose numbers 11-19 into ten ones and some further ones** • using objects • using drawings • written representation (drawing ***or*** equation) **Decompose numbers 11-19 into ten ones and some further ones:** • using objects • using drawings • written representation (drawing ***or*** equation)

- Retrieved from https://salkeiz-cia.orvsd.org/sites/salkeiz-cia.orvsd.org/files/KCC_PlaceVal_Deconst.pdf

Video Lesson Example

https://www.teachingchannel.org/videos/kindergarten-counting-cardinality-lesson

- How did the teacher make connections to the students' prior learning?

- How were the students encouraged to share and learn from each other?

- What supports does Ms. Lassiter use to further her students' thinking?

Group Activity: Plan a learning activity

Count off from one to six. Get into a group with your number. Thinking about everything we have talked about so far, plan a learning activity to teach teen numbers. Your group number's task is below.

Group 1: Demonstrate a lesson using subitizing and ten frames with teen numbers

Group 2: Demonstrate a lesson on number representations for the teen numbers

Group 3: Demonstrate a number talk for teen numbers

Group 4: Demonstrate a think aloud for decomposing a teen number

Group 5: Demonstrate a lesson teaching decomposing teen numbers with the CRA progression

Group 6: Demonstrate an addition lesson using teen numbers

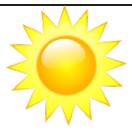
Key Idea #7

Developmental progressions refer to sequences of skills and concepts that children acquire as they build math knowledge.

- Retrieved from https://ies.ed.gov/ncee/wwc/pdf/practice_guides/early_math_pg_111313.pdf

Clear or Cloudy?

Think about the ideas we have discussed. What is clear to you? What can you use in your classroom in August? What ideas are still a little cloudy? Use the space below to list your thoughts and leave them by the door. We will talk about any "cloudy" ideas after the break.

Clear 	Cloudy 

- Retrieved from http://blog.teacherspayteachers.com/5-fast-ways-close-a-lesson/

Group Activity: Teen Number Scoot

Materials needed: double ten frames, manipulatives and counting mats, base ten blocks, dry erase boards and markers (or pencil and paper), number cards for the teen numbers you are working on

Directions:

1. Put a number card on each table, along with the materials to make that number in the following ways:
    - double ten frame and counteten
    - counting manipulatives on a mat
    - with base ten blocks (rods and units)
    - writing up to that number
2. Divide students into groups. At each table they have to make the number on the double ten frame, count out that many manipulatives on the mat, use the base ten blocks to make the number, and write up to that number from one.
3. Check each table as they finish.
4. Rotate groups so all groups get to every number.

*You can start with 11-15, and then move on to 16-20 when ready.

These are resources you can print and use for the teen number scoot activity, if you don't have access to manipulatives.

Printable double ten frame
http://www.mathwire.com/templates/double10framemat.pdf

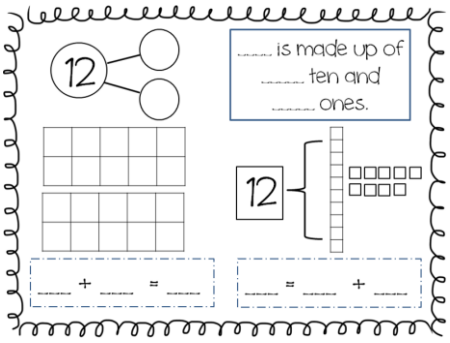
Printable base 10 blocks
http://domathtogether.com/wp-content/uploads/2013/06/Printable-Base-ten-blocks.pdf

Printable number cards
http://esgidocs.esgisoftware.com/Number+Recognition+Activity.pdf

Printable manipulatives
https://www.teachervision.com/tv/printables/scottforesman/Math_K_TTT_12.pdf

Key Idea #8

When you have students practice composing and decomposing teen numbers, you are laying the foundation for teaching place value.



Resources for Teaching Numbers in Base Ten

Virtual Manipulatives

http://nlvm.usu.edu/en/nav/category_g_1_t_1.html

Harry Kindergarten songs—YouTube

https://www.youtube.com/watch?v=1W5aYi3lkho

https://www.youtube.com/watch?v=uedvwH6Ay18

Interactive Online Games

http://www.mhschool.com/math/mathconnects/assets/mhln/00139857/00139857.swf

http://www.mhschool.com/math/mathconnects/assets/asset_view.html?s=CJ_K_07A&type=CJ

http://www.mhschool.com/math/mathconnects/assets/asset_view.html?s=SEC_K_07C&type=SEC

Literature Connection

Meet the Teens by Marcie Cooper

The teen numbers are moving in, right next to 10, 11, and 12! The book has great rhymes to help students remember the names of the teen numbers.

Chicka Chicka 1 2 3 by Bill Martin, Jr.

This book is a companion book to *Chicka Chicka Boom Boom*. The numbers are climbing up the apple tree. It names numbers up to 20, and then counts by tens to 100.

Vocabulary Connection for Numbers in Base Ten

- **tens**—a group of 10 items counted as a whole
- **ones**—items counted individually
- **rod**—base 10 block that equals 10
- **unit**—base 10 block that equals one
- **compose**—putting parts together to make a number
- **decompose**—breaking a number into parts

Can you think of other math vocabulary that students need to know?

Listening and Speaking Connection

How can you incorporate listening and speaking skills into the activities shared today? What questions can you ask students? What type of responses should you hear?

Possible Teacher Questions	Possible Student Responses
Can you show me a group of 15?	This is 10, and I have five more.
How many is 14?	It is a group of 10 and four more.
Why do I need two 10 frames to show the teen numbers?	The numbers all have a 10 and some more ones.
Can you count this group of blocks for me? (13)	Students points to the rod and says 10, then points to the units and counts on, 11, 12, 13.

What other questions could you ask?

Writing Connection: Math Journals

- After reading the book *Meet the Teens*, have students write a story about one of the teen numbers.
- Tell students to divide their journal paper into four sections and give them a teen number to represent four different ways: pictures, equation, word, tally marks, 10 frames, etc.
- Have students write a story problem using teen numbers.

Putting it all together with Instructional Tasks

Using Instructional Tasks

Research indicates that student learning is increased when teachers regularly engage students in cognitively demanding tasks. Teachers must recognize, however, that there are many factors that contribute to the overall effectiveness of using high level tasks with their students. Teachers should carefully select tasks and use them appropriately with their students. Below is a series of questions for teachers to consider when using instructional tasks in their classroom.

1. What are the mathematical goals for using this task?

- Which standards does the task target?
- Which Standards for Mathematical Practices will my students experience by working on this task?
- How does this task build upon prior knowledge?
- How will I know that students have met the mathematical goals?

2. Have I worked the task and anticipated possible solution paths that my students may take?

- Do solution paths include a variety of representations and/or strategies?
- What are common misconceptions students may have when working on the task?

3. How will I maintain the cognitive demand of the task?

- Does the task have multiple entry points to provide access for all learners?
- Does the task allow my students to explore, investigate, and make sense of mathematical ideas on their own?
- Does the task provide personal challenge and productive disequilibrium for all students?
- Am I using Accountable Talk to assure that all students' voices are heard and are actively engaged in the task?
- How will I respond when students struggle or have difficulty starting the task?
- How can I advance the thinking of students who finish the task early?
- Are students required to justify and defend their solutions?
- What questions will I ask students to assess their learning and advance them toward the mathematical goals?

4. How can I summarize the learning during the whole group discussion?

- How will I select and sequence the solution paths to leverage and advance student thinking about the mathematical goal(s) of the lesson?
- How will I help students make connections among students' solution paths and mathematical goal(s)?

Teachers using high-level instructional tasks in their classrooms should follow the launch–explore -summarize lesson structure. It is recommended that teachers give students a regular diet of high level tasks, while also recognizing that there are other productive mathematical activities to use with students.

K.CC.B.4 *The Napping House* Task

This particular task helps illustrate Mathematical Practice Standard 1, Make sense of problems and persevere in solving them. Kindergartners are exposed to multiple problems through the story, *The Napping House*. As students listen to the story they use counters on a ten frame to keep track of each additional person/animal who gets in the bed until the flea bites the mouse. Then the story changes to subtraction as people/animals start to leave the bed. Throughout this guided task, students are introduced to the processes of problem-solving in a non-threatening way. They are able to unpack the parameters of the problem by manipulating the counters one at a time. This allows them to make sense of the actions occurring in the story. These concrete objects help them to conceptualize and solve each problem as posed in the story. The teacher can guide this conceptualization by stopping after each action and asking questions such as,
"What just happened in the story?" "How are we going to show that on our ten frames?" "How many are in the bed now?" and "How do you know how many are in the bed?"

Materials Needed

- ✓ *The Napping House* by Audrey Wood
- ✓ One ten-frame for each child
- ✓ 6-10 counters per child

Directions

- ✓ The teacher reads *The Napping House* to the class, stopping each time a person or animal gets into the bed so the students can add a counter to the ten-frame.

- ✓ After each page, stop to ask the children how many are sleeping in the bed after each counter is added to the ten-frame. For example, at the beginning there should be 1 counter for granny. When the child gets in the bed, there should be 2 counters.

- ✓ Have the children tell how many people there are in the bed now. Do this after each counter is added to the ten-frame. There should be 6 counters on the ten-frame once the wakeful flea is added.

- ✓ Once the flea bites the mouse, the children should begin taking the counters off the ten-frame to represent how many people/animals are still in the bed. For example, once the flea bites the mouse, there are only 4 people/animals left in the bed.

- ✓ By the end of the story, there should be no counters on the ten-frame.

Task Purpose

The purpose of the task is for students to use the context of *The Napping House* to connect counting and cardinality. The teacher or students could also write a simple equation on the classroom dry-erase board each time another person or animal gets into the bed. This would connect counting to addition and subtraction for the students, and would connect with standard K.OA.1. For example, when the child gets in with granny, the equation would be 1+1=2 and so on for each animal added to the bed.

Students can be given 6 counters or 10 counters, depending on their ability level. Students who need more scaffolding should be given 6 counters so that they cannot accidentally miscount and place too many counters on the ten-frame. However, students who are proficient counters should be given 10 counters. This will make them responsible for accurately keeping the count.

Retrieved from https://www.illustrativemathematics.org/content-standards/tasks/1149

K.CC More and Less Handfuls Task

Materials Needed

- ✓ A variety of manipulatives for counting
- ✓ Student recording sheet (see setup)

Setup

- ✓ On a sheet of plain paper write the following sentence frame at the bottom:

I have ____counters. I have _____ (more than/less than/ the same as) my partner. My partner has_____ counters.

- ✓ Copy one sheet per student.
- ✓ Set out a variety of math manipulatives at each table group.
- ✓ Have students work in pairs.

Directions

- ✓ Each student grabs two handfuls of counters.
- ✓ The student combines his/her handfuls into one collection and then counts them.
- ✓ The student then draws and records the quantity on a student-recording sheet.
- ✓ Student partners then complete the sentence frame at the bottom of the page together, stating how many each person had and if they have more or less than their partner.
- ✓ If students are having a hard time with the quantity generated from two handfuls have them only grab one handful of counters to start with.
- ✓ Watch the student's counting strategies. Students should have an organized method for keeping track of items they have already counted. They may use a "pull-off" strategy, moving items from one side of the table to the other, or line them up in a straight line. If students are struggling give them a large paper plate and have them count the items by moving them onto the plate and check it by moving them off the plate.
- ✓ Have the students' progress to grabbing one handful each of two different types of counters in order to ensure they can combine unlike groups into a single collection.

Solution

- ✓ Students should have a pictorial representation of the collection of counters as well as a completed sentence frame such as; "I have 9 counters. I have more than my partner. My partner has 7 counters."
- ✓ Begin with larger types of manipulative such as counting bears, cubes, etc. so that the handfuls do not contain as many items. Have students grab two handfuls from the same type of counters and then vary it so that they grab one handful each from two different types of counters. Progress to smaller counters like buttons or two color counters to increase the quantity that they are dealing with because students will be able to grab larger amounts.

Math Task Resources

https://www.illustrativemathematics.org/content-standards/K

http://commoncoretasks.ncdpi.wikispaces.net/K+Tasks

http://www.insidemathematics.org/common-core-resources/mathematical-content-standards/standards-by-grade/kindergarten

Task Analysis Guide

Lower-Level Demands	***Higher-Level Demands***
<u>Memorization Tasks</u> • Involves either producing previously learned facts, rules, formulae, or definitions, OR committing facts, rules, formulae or, definitions to memory. • Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. • Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.	<u>Procedures With Connections Tasks</u> • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. • Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. • Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.
Procedures Without Connections Tasks • Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. There is little	***Doing Mathematics Tasks*** • Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).

ambiguity about what needs to be done and how to do it. • Have no connection to the concepts or meaning that underlie the procedure being used. • Are focused on producing correct answers rather than developing mathematical understanding. • Require no explanations, or explanations that focus solely on describing the procedure that was used.	• Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships. • Demands self-monitoring or self-regulation of one's own cognitive processes. • Requires students to access relevant knowledge and experiences and make appropriate use of them in working through the task. • Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions. • Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.

Mathematics Teaching in the Middle School. Also in: Stein, Smith, Henningsen, & Silver (2000). Implementing standards-based mathematics instruction: A casebook for professional development, p. 16. New York: Teachers College Press.

Kindergarten Instructional Tasks

All instructional materials previously available on TNCore.org are now hosted at eduToolbox.org. Kindergarten resources can be found here: http://edutoolbox.org/tntools/list/grade/819/955/0#958

- **The Seashell Collections** (K.CC.B.4, K.CC.B.4a, K.CC.B.4b, K.CC.B.4c, K.CC.B.5, K.CC.C.6)
- **The Four-Leaf Clover** (K.OA.A.1, K.OA.A.2, K.OA.A.4)
- **Counting Balls** (K.CC.B.4, K.CC.B.4a, K.CC.B.4b, K.CC.B.4c, K.CC.B.5)
- **Sammy's Pets** (K.OA.A.1, K.OA.A.3)

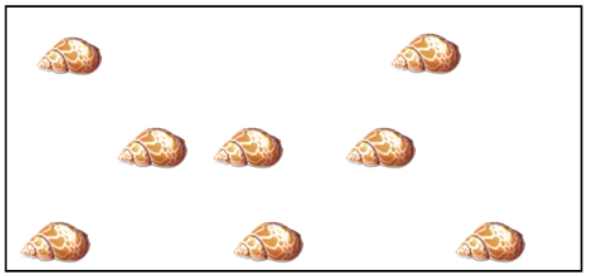
Task: The Seashell Collections	Kindergarten

Carrie, Ashley, and Suzanne are collecting seashells. Each collection is shown below.

Carrie's Seashell Collection

Ashley's Seashell Collection

Suzanne's Seashell Collection



Count the number of shells in each girl's collection. How many shells are in each collection?

Suzanne thinks she has the most seashells. Do you agree or disagree with Suzanne? Explain your answer.

Teacher Notes:

A student page is provided so that students may have easy access to count the number of shells in each collection. The seashell collections contain the same number of shells but are arranged in a line, a rectangular array, and a scattered configuration. Students should understand that the arrangement of the objects in a set does not affect the number of objects in the set. Students should also realize that another important part of counting accurately is being able to keep track of what has already been counted and what remains to be counted.

If students should have trouble understanding that all of the seashell collections contain the same number of shells, you might suggest that they cover the seashells with counters, using a different color for each girl. Students can then determine that the number of objects is the same by matching the counters with a one-to-one correspondence or rearranging the counters to make 3 identical sets with the same arrangement.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
K.CC.B.4 Understand the relationship between numbers and quantities; connect counting to cardinality. **K.CC.B.4a** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **K.CC.4b** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **K.CC.4c** Understand that each successive number name refers to a quantity that is one larger. **K.CC.B.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects. **K.CC.C.6** Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1] [1] Include groups with up to ten objects.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings	
• Counting tells how many things are in a set. When counting a set of objects, the last word in the counting sequence names the quantity for that set. (Van de Walle) • Counting includes one-to-one correspondence, regardless of the kind of objects in the set and the order in which they are counted. (NCTM) • The relationship between one quantity and another can be an equality or inequality relation. (NCTM)	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Student accurately counts the number of shells in each seashell collection and determines that each collection has eight shells. Student recognizes that the number of shells in each collection is equal and Suzanne is not correct that she has the most seashells.	Assessing Questions • How did you count the number of shells in each collection? • How do you know there are eight seashells in each collection? • How do you know the number of seashells in each collection is the same? Advancing Questions • How is it possible that each collection has eight shells when the collections look so different? • What if each collection contained different kinds of shells, would they still have the same number?
Student counts the number of shells in each collection. Student matches the number of shells in each collection using one-to-one correspondence to determine that the number of shells in each collection is the same. Student then determines that Suzanne is not correct that she has the most seashells.	Assessing Questions • How did you count the number of shells in each collection? • How many seashells are in each collection? Advancing Questions • How is it possible that each collection contains the same number of shells when the collections look so different? • If each set contain eight shells, are they equal? How do you know?
Possible Student Misconceptions	
Student does not accurately count the number of shells in each collection.	• How did you determine the number of shells in each collection? • Count the shells again. Did you get the same number?
Student does not understand that the number of shells in each collection is the same. Student thinks that Suzanne's collection contains more because the collection is scattered and covers a greater area.	• How many shells are in each collection? • How does the number of shells in the collection help us determine who has the most shells? • Does the arrangement of the shells affect the number of shells in the collection? Why or why not?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Assessing Questions • How can you determine the number of shells in each collection? • What number begins the counting sequence?
If students finish early….	Assessing Questions • How many more shells does each girl need to have a total of 10 shells in her collection? • How many shells do the girls have altogether?

Discuss/Analyze
Whole Group Questions
• How did you determine the number of seashells in each collection? • Did the arrangement of the seashells make a difference or change the number of shells in each collection? • How did you determine whether or not Suzanne had the most shells?

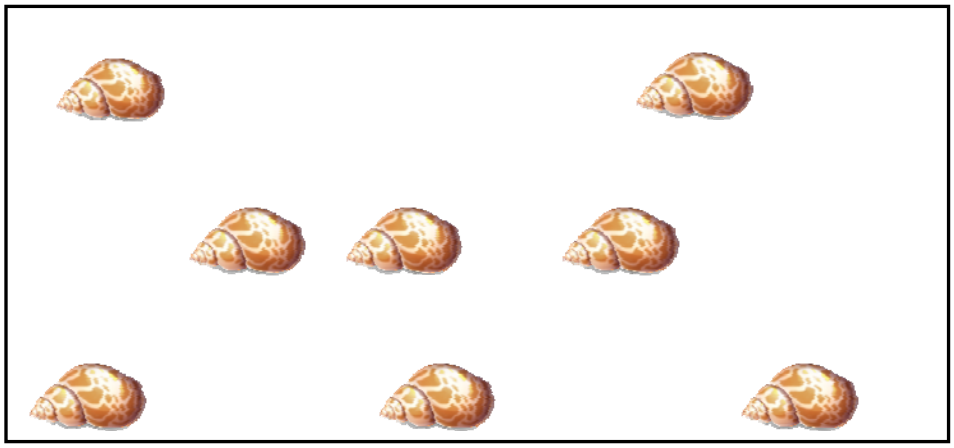
The Seashell Collections Task

Carrie, Ashley, and Suzanne are collecting seashells. Each collection is shown below.

Carrie's Seashell Collection

Ashley's Seashell Collection

Suzanne's Seashell Collection



Count the number of shells in each girl's collection. How many shells are in each collection?

Suzanne thinks she has the most seashells. Do you agree or disagree with Suzanne? Explain your answer.

Task: The Four-Leaf Clover Task	Kindergarten

Max is collecting four-leaf clovers. He found 5 clovers while playing at the park. He found 2 clovers in his yard. How many four-leaf clovers does Max have? Use a drawing, objects, or a number sentence (equation) to show how you found your answer.

Max claims that if he finds 3 more clovers he will have 10. Do you agree or disagree with Max? Use a drawing or a number sentence (equation) in your explanation.

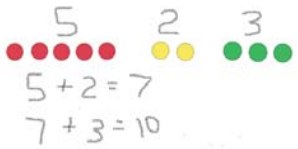
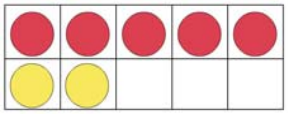
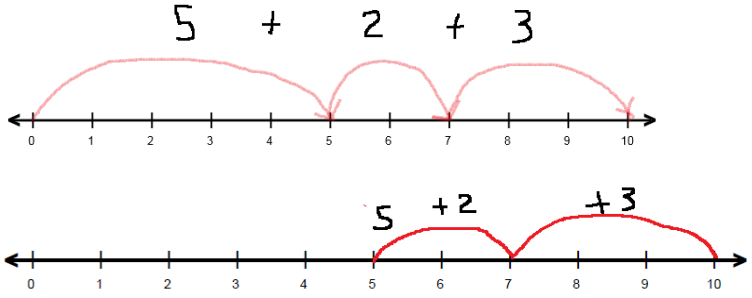
Teacher Notes:

Cubes, counters, or other manipulatives should be available for students to use as needed. A part-part-whole map or a tens frame may be helpful for some students to visualize and to make sense of the problem. The term "number sentence" is used instead of "equation". Teachers should model the term "equation" but students may continue to use the term "number sentence". Students may choose not to write an equation, but should be able to explain how they found the answer with a drawing or model. If students do not write an equation, the teacher may choose to model this in the whole group discussion.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings[1], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.A.2** Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem. **K.OA.A.4** For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation. [1] Drawings need not show details, but should show the mathematics in the problem.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

- The addition of whole numbers is based on sequential counting.
- Addition equations can be used to describe situations that involve combining quantities.

Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Direct Modeling with manipulatives or drawings Student draws or counts 5 objects and 2 objects and recognizes that the quantities should be combined to get a total of 7.  Student then determines that 7 and 3 more will equal 10. Student agrees with Max's statement that he needs 3 more to have a total of 10. Using a tens frame, student can count that 5 and 2 are 7.  Student agrees that if Max had three more he would have a total of ten.	**Assessing Questions** • How did you determine the number of clovers that Max has? • How did you determine the number of clovers that Max will need to have 10 clovers? • Describe your drawing or equation. How did this help you determine your solution? • What do the 5 and 2 represent in your equation? **Advancing Questions** • You have determined that Max has 7 clovers. How can you determine how many more he needs to have 10? • Can you write an equation to represent this situation? (for students who did not write an equation)
Counting On Students begins with 5 (or counts up to 5) and counts on 2 more for a total of 7 and then counts on 3 more for a total of 10. Student then agrees with Max's statement that if he had 3 more he would have a total of 10.  This could be illustrated with a number line in one of the following ways:  	**Assessing Questions** • How did you determine the number of clovers that Max has? • How did you determine the number of clovers that Max will need to have 10 clovers? • Where did you start counting? How do you know where to stop counting? • What do the 5 and 2 represent in your equation? **Advancing Questions** • You have determined that Max has 7 clovers. How can you determine how many more he needs to have 10? • Can you write an equation to represent this situation? (for students who did not write an equation) • How can a number line be used to model this situation?

Possible Student Misconceptions	
Student does not correctly add the number of clovers and finds an incorrect sum.	• How do you know where to start counting? • What does the starting number represent? • How much will you add to the starting number? • What do the numbers 5, 2 and 3 represent? • How can counting help you find your answer?
Student does not agree that if Max finds 3 more clovers then he will have 10.	• How many clovers does Max have after he finds 5 and 2 clovers? • If Max has 7 clovers, how many more will he need to have 10?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	• Describe the story problem. • How many clovers does Max have after he visits the park and his yard? • How can you model this story problem with cubes?
If students finish early….	• How many more clovers will Max need to find so that he has 12 clovers? • If Max finds 5 clovers and 2 clovers, but then loses 1 clover. How many more will he need to have 10 clovers?
Discuss/Analyze	
Whole Group Questions	
• Describe how you found the total number of clovers that Max found at the park and in his yard? • How can a tens frame, cubes or a number line help us model this problem? • How can we represent this problem with an equation?	

The Four-Leaf Clover Task

Max is collecting four-leaf clovers. He found 5 clovers while playing at the park. He found 2 clovers in his yard. How many four-leaf clovers does Max have? Use a drawing, objects and or a number sentence to show how you found your answer.

Max claims that if he finds 3 more clovers he will have 10. Do you agree or disagree with Max? Use a drawing or a number sentence in your explanation.

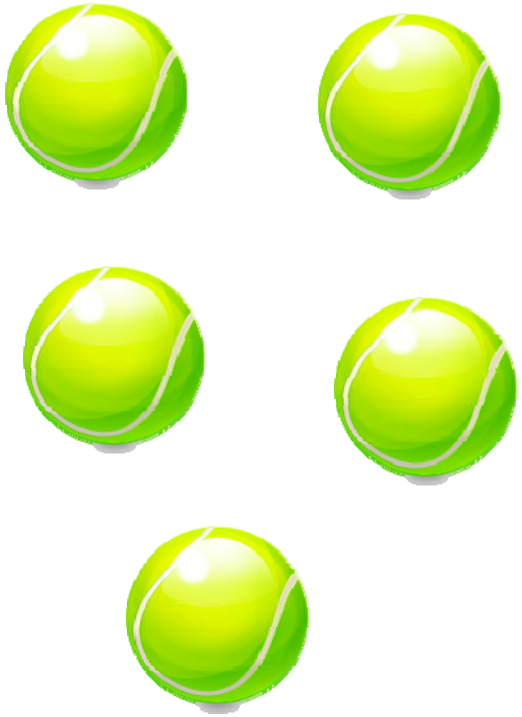
Task: Counting Balls	**Kindergarten**
Zanna and Sage like to play with different kinds of sports balls. Zanna has 5 tennis balls and Sage has 5 golf balls. Zanna says that she has more balls because the tennis balls are bigger. Do you think Zanna is right? Explain your answer with words and pictures. (See e below for pictorial representation of the size of the balls)	
Teacher Notes:	
Use of actual tennis balls and golf balls or cut outs of the attached figures as manipulatives is recommended. Conservation of number is essential to conceptual understanding of counting.	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
Count to tell the number of objects **K.CC.B.4** Understand the relationship between numbers and quantities, connect counting to cardinality. **K.CC.B.4a** When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object. **K.CC.B.4b** Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted. **K.CC.B.4c** Understand that each successive number name refers to a quantity that is one larger. **K.CC.B.5** Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle or as many as 10 things in a scattered configuration; given a numb3er from 1-20 count out that many objects.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings	
• Counting includes one-to-one correspondence, regardless of the kind of objects in the set and the order in which they are counted. • When counting objects in a group/set, the last number stated names the total number of objects in that group/set.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Student will count each set of balls and label or name them with the correct number. Student will then state that because there is the same number of tennis balls that there is of golf balls Zanna and Sage have the same or equal groups so Zanna's claim is incorrect.	Assessing Question: How did you decide that Zanna was not right? What strategy did you use to determine whether Zanna had more? Advancing Question: What would you change in the problem to make Zanna's claim correct? Does the size of the objects make a difference in this problem? Is there another way to show /explain your answer?

Student may line the balls up using one-to-one correspondence to show that the groups/sets of tennis balls and golf balls are the same. Zanna's claim is incorrect.	Assessing Question: How did you know to line the balls up? Did that make it easier? Why or why not? Advancing Question: What would you change in the problem to make Zanna's claim correct? Does the size of the objects make a difference? Is there another way to show /explain your answer?
Student may use the pictorial representation to connect one tennis ball to one golf ball until all are paired and relate that the girls have the same number.	Assessing Question: Tell me how you decided to draw the lines to match a tennis ball and a golf ball. Tell me how matching a tennis ball with a golf ball helped you answer the question. Advancing Question: If each girl had some of the tennis balls and some of the golf balls could you use the same strategy? Explain your answer.
Possible Student Misconceptions	
Students may think that there are more tennis balls because they are larger than the golf balls. When placed in a line, the line of tennis balls is longer than the line of golf balls so it may appear there are more. Students may count the balls in random order and count some of them more than once.	Assessing Questions: Why do you think Zanna said she had more? How could you find out? Advancing Question: Is there a way to arrange the balls that would help you? If you move the balls around would that change your answer?
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Assessing Question: Why do you think Zanna said she had more balls? What can you do to see how many balls each girl has? Advancing Question: Is there a way to arrange the balls that would help you see if Zanna is correct?
If students finish early….	Have the students show how each girl could have an equal number of balls but a combination of both tennis balls and golf balls. Write number sentences to demonstrate their answers. If you gave each girl two more balls would they still have an equal number of balls? Write number sentences to demonstrate their answers.

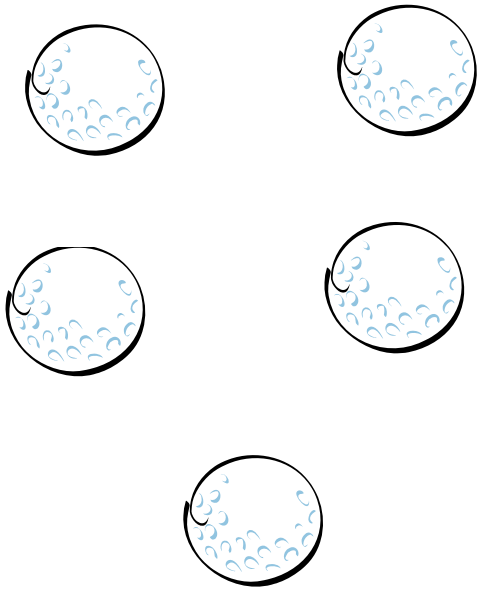
Discuss/Analyze
Whole Group Questions
• What ways did you use to count the balls? • How were the counting strategies that we shared similar and different? • How did you know if you had already counted a ball? How did you keep from counting it more than one time? • Does it matter where you start when you are counting objects? Explain your answer. • When you say the number of the last ball in a group, what does that number tell you? • If you rearrange the balls in a group and count them again, will you get the same number? Explain your answer. • If you are counting things in a group, does it matter if the things are different sizes? Explain your answer.

Tennis Balls and Golf Balls

Zanna's Tennis Balls



Sage's Golf Balls

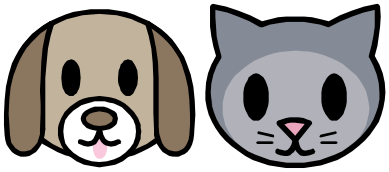


Task: Sammy's Pets	Kindergarten
Sammy has 7 pets. Some are dogs and some are cats. How many dogs and how many cats could Sammy have? Use a drawing and a number sentence (equation) to explain your answer. Choose another way to show how Sammy could have 7 pets if some are dogs and some are cats. Use a drawing and a number sentence (equation) to explain your answer.	
Teacher Notes:	
Cubes or other manipulatives should be available for students to use if needed. *Do not give students 7 cubes.* Have cubes available and allow students to count the number of cubes needed. A part-part-whole map may be helpful for some students to recognize that 7 cubes should be decomposed into two groups and that the two groups have a total of 7 pets. The term "number sentence" is used instead of "equation". Teachers may choose to model the term "equation" but students may choose to continue to use the term "number sentence".	
Common Core State Standards for Mathematical Content	**Common Core State Standards for Mathematical Practice**
K.OA.A.1 Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations. **K.OA.A.3** Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1). [2] Drawings need not show details, but should show the mathematics in the problem.	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.
Essential Understandings	
• A quantity can be decomposed into two parts in more than one way. • A quantity can be decomposed into two parts and the whole quantity remains the same. The decomposition may be recorded as an addition equation.	
Explore Phase	
Possible Solution Paths	**Assessing and Advancing Questions**
Direct modeling with manipulatives: Student counts 7 objects and divides the objects into two groups – one to represent dogs and one to represent cats. (A part-part-	Assessing Questions • Which group of cubes represents the number of dogs and which represents the number of cats? • Why did you start with 7 cubes?

whole map may be helpful for some students.)	• Describe how you found the answer to the problem. Advancing Questions • What are other solutions to this problem? • Is it possible for Sammy to have the same number of cats and dogs? Why or why not?
Counting on from a number less than 7: Student chooses a number less than 7 to represent the number of dogs and then counts on to determine the number of cats needed to make a total of seven pets. Possible representations may include objects or number lines.	Assessing Questions • Describe how you found the answer to the problem. • Why did you start with a number less than 7? Advancing Questions • What are other solutions to this problem? • Is it possible for Sammy to have the same number of cats and dogs? Why or why not?
Known Addition Combinations Student chooses a known combination for seven and recognizes that each addend could represent the number of cats or dogs. Examples could include: 1 + 6 = 7 so Sammy has 1 dog and 6 cats 1 + 6 = 7 so Sammy has 1 cat and 6 dogs 2 + 5 = 7 so Sammy has 2 dogs and 5 cats	Assessing Questions • Describe how you found the answer to the problem. • I notice that you said Sammy could have 1 dog and 6 cats or 6 cats and 1 dog. These use the same numbers. Describe how they are different. Advancing Questions • What are other solutions to this problem? • Is it possible for Sammy to have the same number of cats and dogs? Why or why not?
Possible Student Misconceptions	
Student inaccurately counts the number of cubes in each set or inaccurately counts when adding up to 7.	Do the number of dogs and cats equal 7? How do you know? Do you think it is possible for Sammy to have 8 dogs? Why or why not?
Student decomposes the 7 into more than two parts.	What does each number represent? How could we represent dogs and cats with the cubes? (Perhaps use two colors, etc.)
Entry/Extensions	**Assessing and Advancing Questions**
If students can't get started….	Tell me what you know about Sammy's pets. Show me with cubes the number of pets that Sammy has.
If students finish early….	What is greatest number of dogs or cats that Sammy could have if Sammy has both cats and dogs? How can the equation 5 + 2 = 7 represent two different solutions? What are all of the possible solutions to this problem and how do you know you have found all solutions?

Discuss/Analyze
Whole Group Questions
How can the seven be decomposed into two groups in more than one way? Describe a number sentence (equation) that could be used to show how 7 can be decomposed into 2 parts. Describe the relationship between the number sentence and number of pets Sammy has.

Sammy's Pets Task



Sammy has 7 pets. Some are dogs and some are cats.

How many dogs and how many cats could Sammy have?
Use a drawing and a number sentence to explain your answer.

Choose another way to show how Sammy could have 7 pets if some are dogs and some are cats?
Use a drawing and a number sentence to explain your answer.

Group Project

After reading the task examples, work with your grade level groups to create a Math task for your students.

- ✓ What math practice standards can you address?
- ✓ How can you connect the task to literacy components?
- ✓ What writing skills can you incorporate to the task?

Math Task Template

PART 1: SELECTING AND SETTING UP A MATHEMATICAL TASK	
What are your mathematical goals for the lesson? (i.e., what do you want students to know and understand about mathematics as a result of this lesson?)	
• What are your expectations for students as they work on and complete this task? • What resources or tools will students have to use in their work that will give them entry into, and help them reason through, the task? • How will the students work—independently, in small groups, or in pairs—to explore this task? • How will students record and report their work?	
How will you introduce students to the activity so as to provide access to *all* students while maintaining the cognitive demands of the task?	

PART 2: SUPPORTING STUDENTS' EXPLORATION OF THE TASK	
As students work independently or in small groups, what questions will you ask to— • help a group get started or make progress on the task? • focus students' thinking on the key mathematical ideas in the task? • assess students' understanding of key mathematical ideas, problem- solving strategies, or the representations? • advance students' understanding of the mathematical ideas?	
• How will you ensure that students remain engaged in the task? • What assistance will you give or what questions will you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance is solving the task? • What will you do if a student (or group) finishes the task almost immediately? How will you extend the task so as to provide additional challenge?	

PART 3: SHARING AND DISCUSSING THE TASK	
• How will you orchestrate the class discussion so that you accomplish your mathematical goals? • Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why? • What specific questions will you ask so that students will— 1. make sense of the mathematical ideas that you want them to learn? 2. expand on, debate, and question the solutions being shared? 3. make connections among the different strategies that are presented? 4. look for patterns? 5. begin to form generalizations? • What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas that you intended for them to learn?	

Adapted from: Smith, Margaret Schwan, Victoria Bill, and Elizabeth K. Hughes. "Thinking Through a Lesson Protocol: Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School 14* (October 2008): 132-138.

Reflecting on the Training

Thinking about the topics that have been shared, what will you take back to your school and share with your teammates? List your ideas below.

- **Numeracy-Rich Classroom Components**

- **Subitizing and Number Representations**

- **Math Talk (Number Talks and Think Alouds)**

- **Five and 10 Frames**

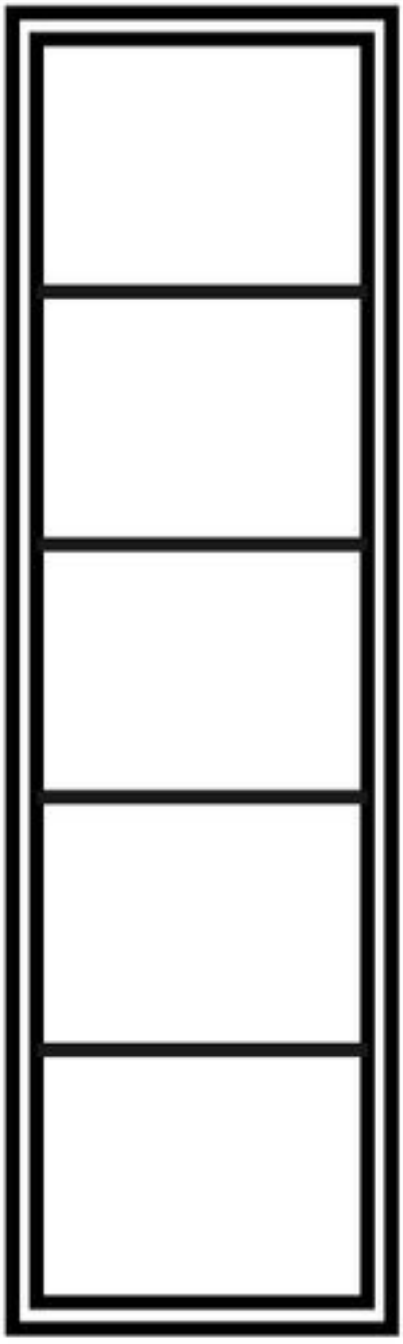
- **Addition and Subtraction**

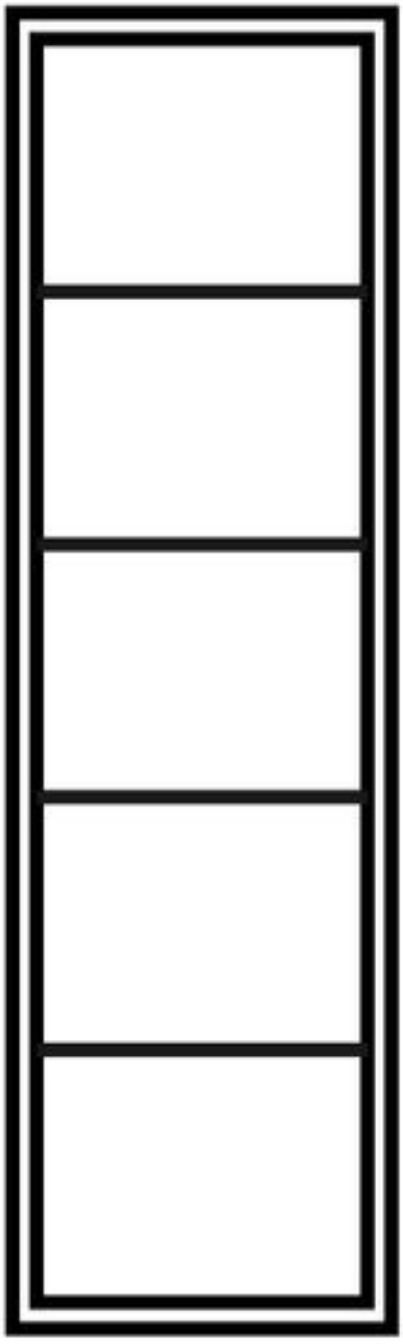
- **Numbers in Base 10**

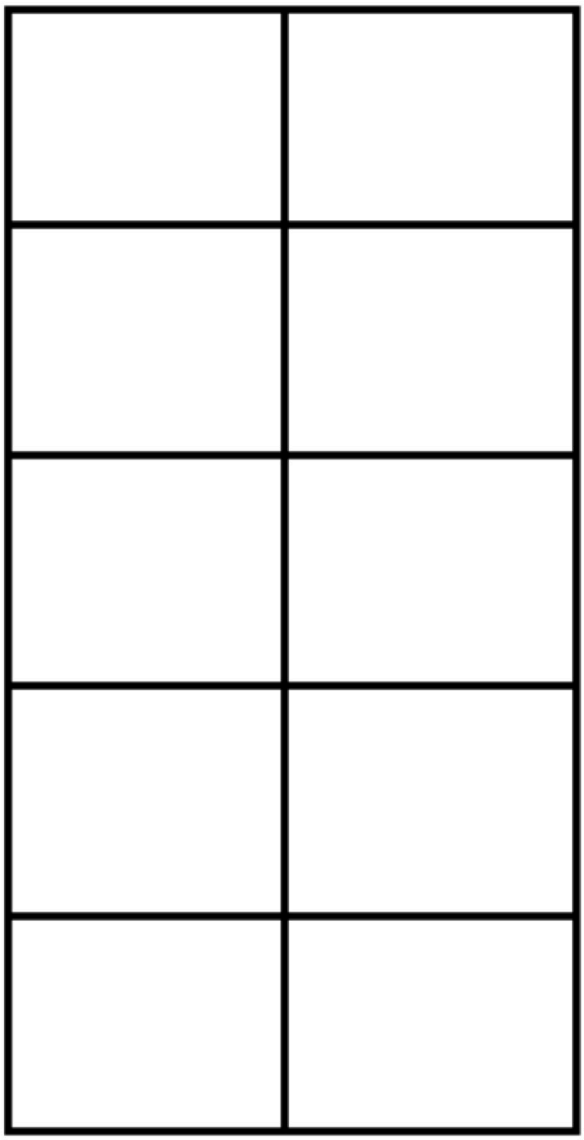
- **Math Tasks**

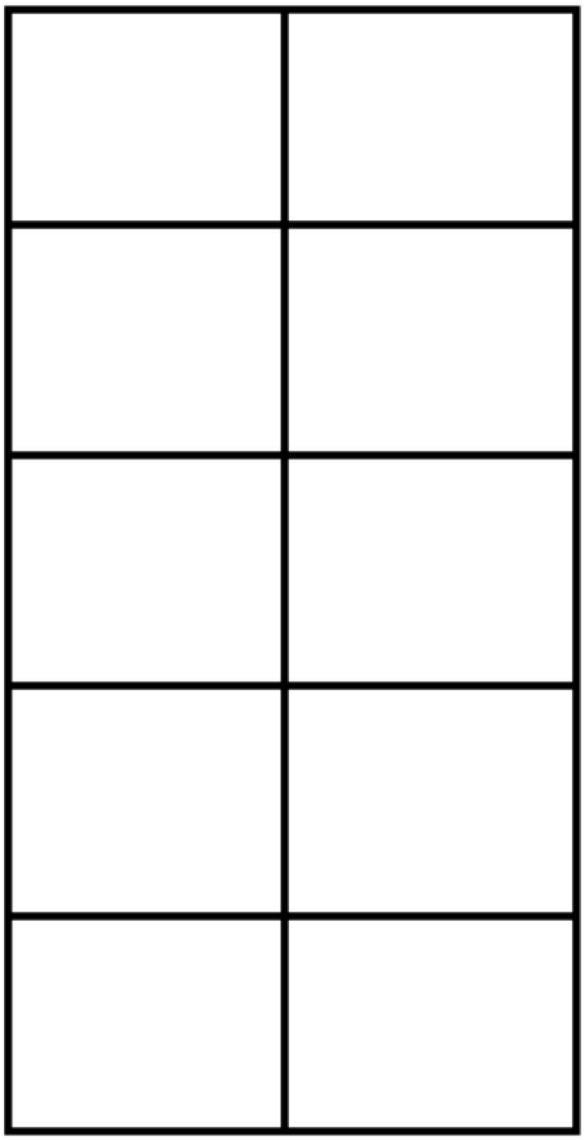
Appendix

TAB









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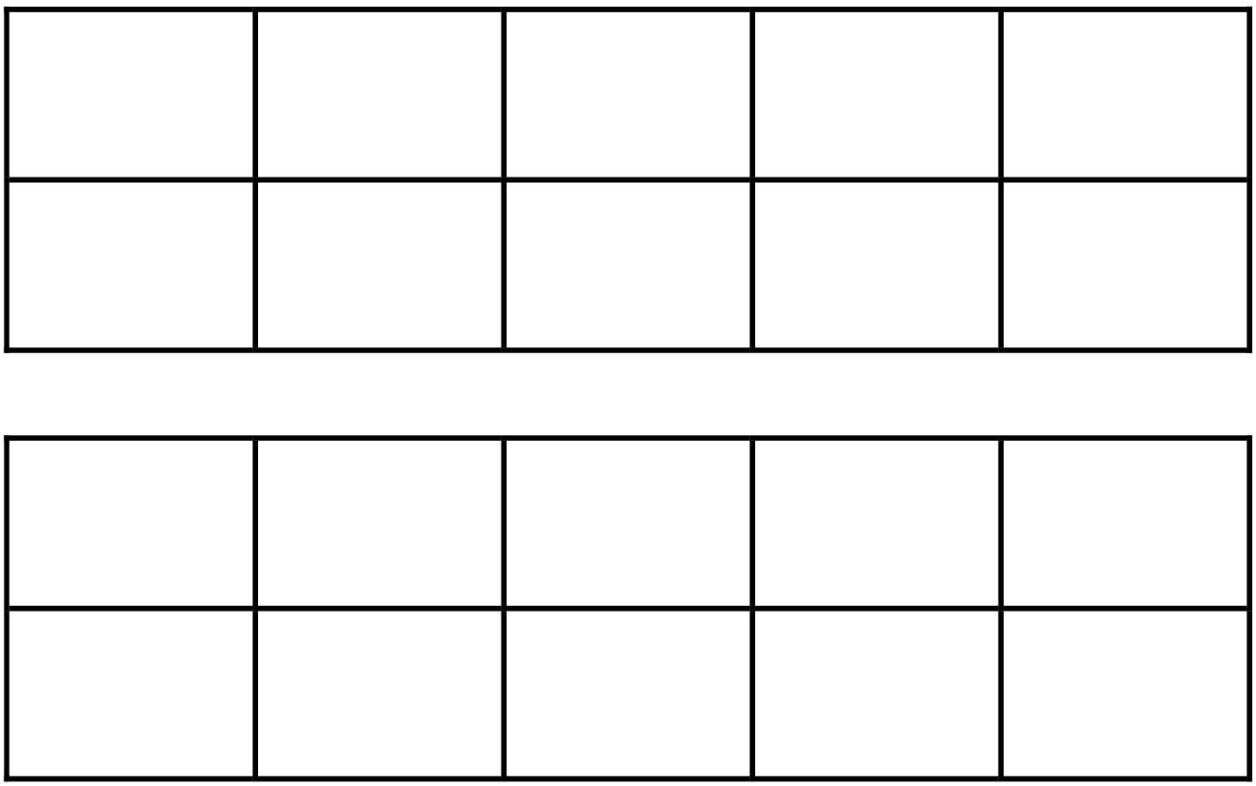

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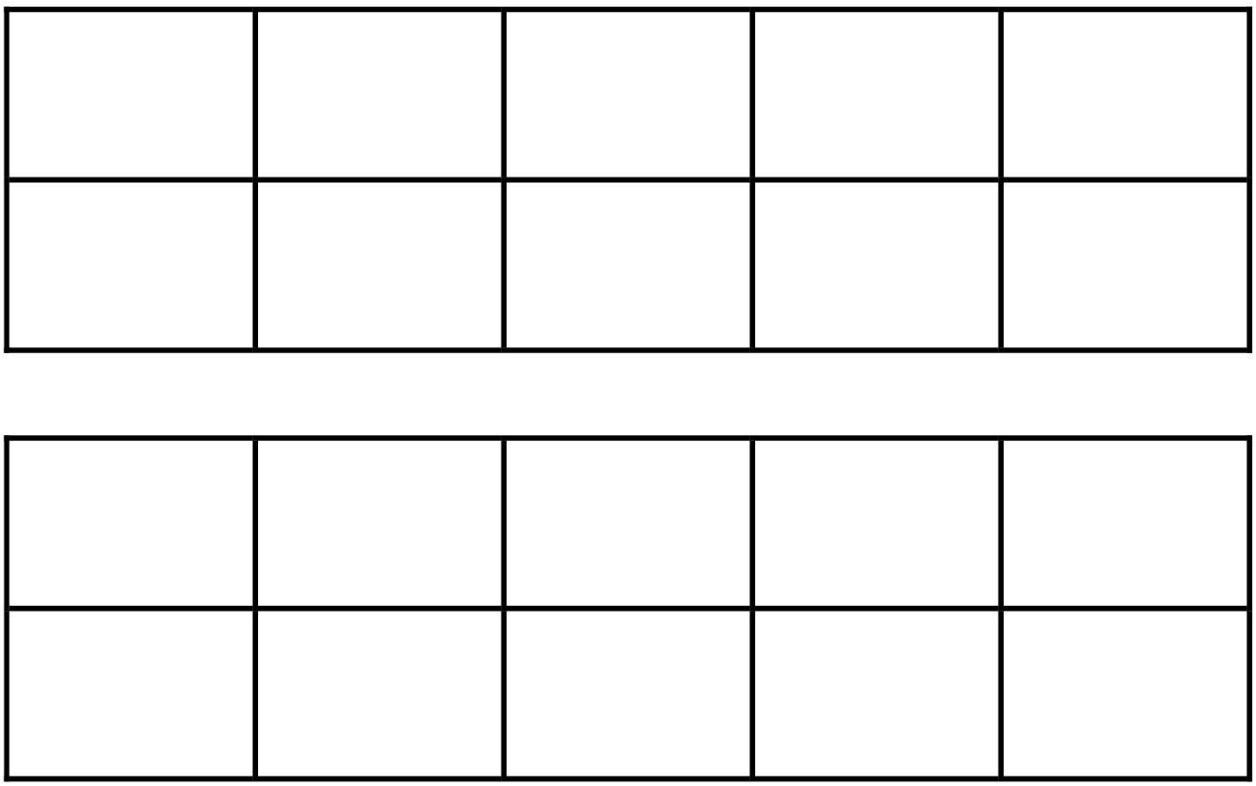

Group Project

After reading the task examples, work with your grade level groups to create a Math task for your students.

- ✓ What math practice standards can you address?
- ✓ How can you connect the task to literacy components?
- ✓ What writing skills can you incorporate to the task?

Math Task Template

PART 1: SELECTING AND SETTING UP A MATHEMATICAL TASK	
What are your mathematical goals for the lesson? (i.e., what do you want students to know and understand about mathematics as a result of this lesson?)	
• What are your expectations for students as they work on and complete this task? • What resources or tools will students have to use in their work that will give them entry into, and help them reason through, the task? • How will the students work—independently, in small groups, or in pairs—to explore this task? • How will students record and report their work?	
How will you introduce students to the activity so as to provide access to *all* students while maintaining the cognitive demands of the task?	

PART 2: SUPPORTING STUDENTS' EXPLORATION OF THE TASK	
As students work independently or in small groups, what questions will you ask to— • help a group get started or make progress on the task? • focus students' thinking on the key mathematical ideas in the task? • assess students' understanding of key mathematical ideas, problem- solving strategies, or the representations? • advance students' understanding of the mathematical ideas?	
• How will you ensure that students remain engaged in the task? • What assistance will you give or what questions will you ask a student (or group) who becomes quickly frustrated and requests more direction and guidance is solving the task? • What will you do if a student (or group) finishes the task almost immediately? How will you extend the task so as to provide additional challenge?	

PART 3: SHARING AND DISCUSSING THE TASK	
• How will you orchestrate the class discussion so that you accomplish your mathematical goals? • Which solution paths do you want to have shared during the class discussion? In what order will the solutions be presented? Why? • What specific questions will you ask so that students will— 1. make sense of the mathematical ideas that you want them to learn? 2. expand on, debate, and question the solutions being shared? 3. make connections among the different strategies that are presented? 4. look for patterns? 5. begin to form generalizations? • What will you see or hear that lets you know that *all* students in the class understand the mathematical ideas that you intended for them to learn?	

Adapted from: Smith, Margaret Schwan, Victoria Bill, and Elizabeth K. Hughes. "Thinking Through a Lesson Protocol: Successfully Implementing High-Level Tasks." *Mathematics Teaching in the Middle School 14* (October 2008): 132-138.

Department of Education

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